Voor eenheid van gewicht

$$\int \rho \psi \, dV = M$$

$$\frac{d(\rho\psi)}{d\rho_x} = \mu_1 \qquad \frac{d(\rho\psi)}{d\rho_y} = \mu_2$$

$$\rho\psi = \mu_1 \rho_x + \mu_2 \rho_y - f(h)$$

$$\left(\frac{d\psi}{dv}\right)_{x\,T_2} = -p$$

$$\psi = \mu_1(1-x) + \mu_2 x - v f(h)$$

$$\psi + pv = \mu_1(1-x) + \mu_2 x$$

$$\rho(\varepsilon - T\eta + gh) = \mu_1 \rho_x + \mu_2 \rho_y - f(h)$$

$$\varepsilon - T\eta + gh = \mu_1(1-x) + \mu_2 x - v f(h)\rho$$

$$d\varepsilon - T d\eta + p dv + v dp + g dh = (\mu_2 - \mu_1) dx$$

h = constant en p dus ook

$$d\varepsilon - T d\eta + p dv = (\mu_2 - \mu_1) dx$$

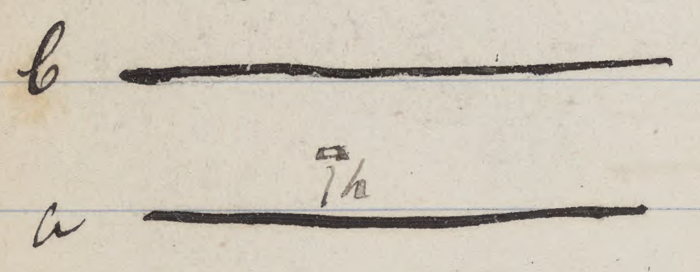
of $dp = -\rho g dh$

DSI

boven b $\quad \varepsilon = -\frac{1}{2}\int_0^\infty \psi(u)\,du\, \frac{\rho_{h-u}+\rho_{h+u}}{2}$

beneden b $\quad \varepsilon = -\frac{1}{2}\int_0^\infty \psi(u)\,du\, \rho_{h+u}$

$\qquad -\frac{1}{2}\int_0^h \psi(u)\,du\, \rho_{h-u}$



na ontwikkeling

$$\varepsilon = -\frac{1}{2}\int_0^\infty \psi(u)\,du\left\{\rho_{h-u}+\rho_{h+u}\right\} + \frac{1}{2}\int_h^\infty \psi(u)\,du\,\rho_{h-u}$$
$$-\int_h^\infty \rho_2\,\psi_2(u)\,du$$

$$\psi = -kT\log\left(\frac{1}{\rho}-b\right) - a\rho - \frac{c_2}{2}\frac{\partial^2\rho}{\partial h^2} - \int_h^\infty \rho_2\,\psi_2(u)\,du$$
$$+\frac{1}{2}\rho\int_h^\infty \psi(u)\,du - \frac{1}{2}\frac{d\rho}{dh}\int_h^\infty u\,\psi(u)\,du + \frac{1}{4}\int_h^\infty u^2\psi(u)\,du\,\frac{\partial^2\rho}{\partial h^2}$$

$$\delta\int\rho\,\psi\,dh = \int\delta\rho\,dh\left\{\frac{\partial\rho\left[-a\rho-kT\log\left(\frac{1}{\rho}-b\right)\right]}{\partial\rho} - \frac{c_2}{2}\frac{\partial^2\rho}{\partial h^2} - \int_h^\infty \rho_2\,\psi_2(u)\,du\right.$$
$$+\rho\int_h^\infty \psi(u)\,du - \frac{1}{2}\frac{d\rho}{dh}\int_h^\infty u\,\psi(u)\,du + \frac{1}{4}\int_h^\infty u^2\psi(u)\,du\,\frac{\partial^2\rho}{\partial h^2}$$
$$\left. - \frac{d\left(-\frac{1}{2}\rho\int_h^\infty u\,\psi(u)\,du\right)}{dh} + \frac{d^2\left\{-\frac{c_2}{2}\rho + \frac{1}{4}\rho\int_h^\infty u^2\psi(u)\,du\right\}}{dh^2}\right\}$$

of
$$K = \frac{\partial\rho\left[-a\rho-kT\log\left(\frac{1}{\rho}-b\right)\right]}{\partial\rho} - c_2\frac{\partial^2\rho}{\partial h^2} - \int_h^\infty \rho_2\,\psi_2(u)\,du + \rho\int_h^\infty \psi(u)\,du$$
$$+\frac{1}{2}\rho\,\frac{d\int_h^\infty u\,\psi(u)\,du}{dh} + \frac{1}{2}\frac{\partial^2\rho}{\partial h^2}\int_h^\infty u^2\psi(u)\,du + \frac{1}{4}\rho\,\frac{d^2\int_h^\infty u^2\psi(u)\,du}{dh^2}$$
$$+\frac{1}{2}\frac{d\rho}{dh}\,\frac{d\int_h^\infty u^2\psi(u)\,du}{dh}$$

$$K\rho + L = \rho\left\{-a\rho - kT\log\left(\frac{1}{\rho}-b\right)\right\}p - \frac{c_2}{2}\left(\frac{\partial\rho}{\partial h}\right)^2 - \rho\rho_2\int_h^\infty \psi_2(u)\,du + \rho_2\int_h^\infty \rho\,\psi_2(u)\,du$$
$$+\frac{1}{2}\rho^2\int_h^\infty \psi(u)\,du - \frac{1}{2}\int_h^\infty \rho^2\psi(u)\,du + \frac{1}{2}\int\rho\,d\rho\,\frac{d\int_h^\infty u\,\psi(u)\,du}{dh}$$
$$+\frac{1}{2}\int d\rho\,\frac{d^2\rho}{dh^2}\int_h^\infty u^2\psi(u)\,du + \frac{1}{4}\int\rho\,d\rho\,\frac{d^2\int_h^\infty u^2\psi(u)\,du}{dh^2} + \frac{1}{2}\int\left(\frac{d\rho}{dh}\right)^2 dh\,\frac{d\int_h^\infty u^2\psi(u)\,du}{dh}$$

~~h groot~~

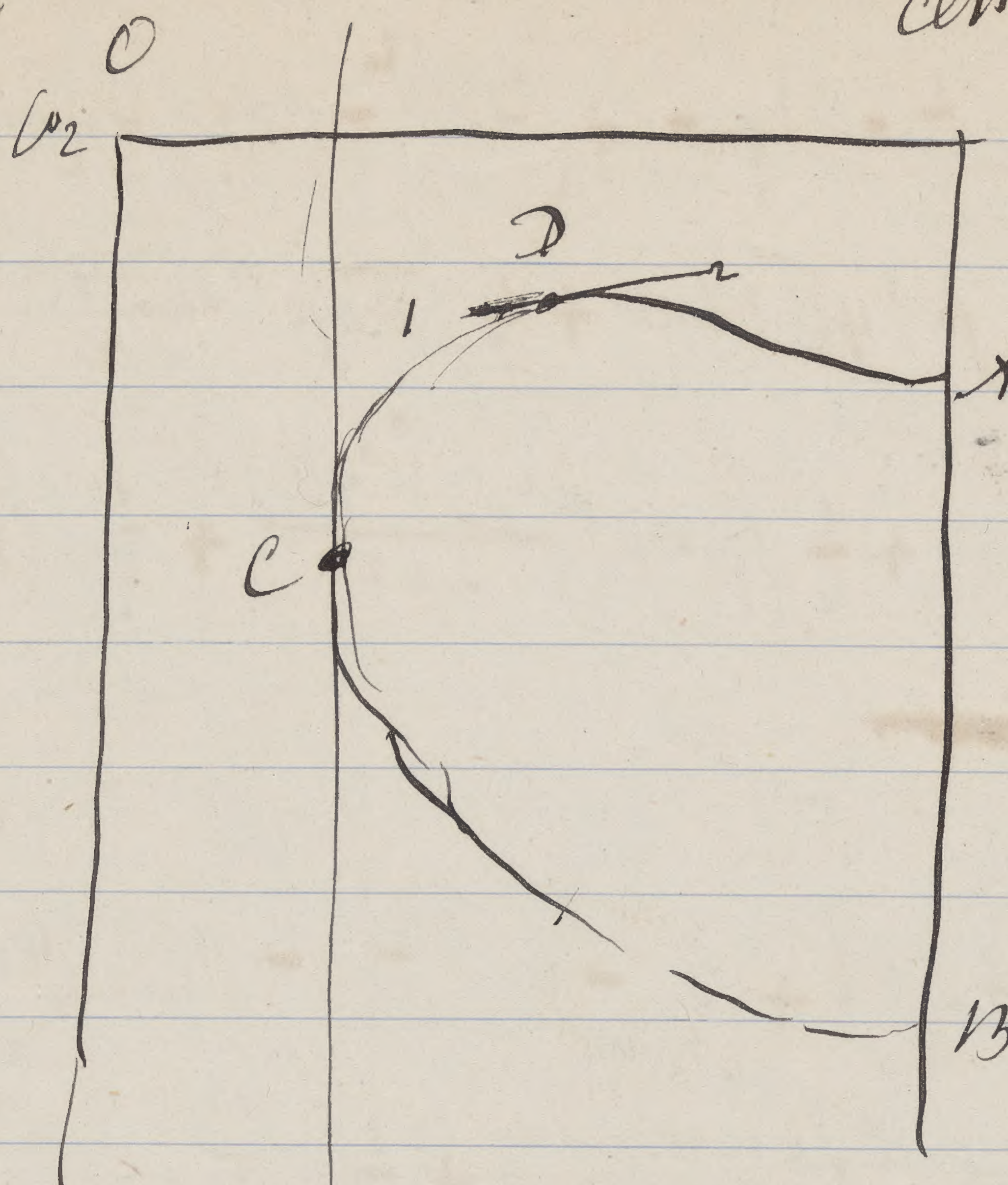
~~$K\rho_1 + L =$~~

$$K\rho = \rho\left\{-a\rho - kT\log\left(\frac{1}{\rho}-b\right)\right\} + p_\rho - c_2\rho\frac{\partial^2\rho}{\partial h^2} - \rho\rho_2\int_h^\infty \psi_2(u)\,du.$$
$$+\rho^2\int_h^\infty \psi(u)\,du + \frac{1}{2}\rho^2\,\frac{d\int_h^\infty u\,\psi(u)\,du}{dh} + \frac{1}{2}\rho\frac{\partial^2\rho}{\partial h^2}\int_h^\infty u^2\psi(u)\,du$$
$$+\frac{1}{4}\rho^2\,\frac{d^2\int_h^\infty u^2\psi(u)\,du}{dh^2} + \frac{1}{2}\rho\frac{d\rho}{dh}\,\frac{d\int_h^\infty u^2\psi(u)\,du}{dh}$$

$$L = -p_\rho - c_2\left\{\frac{1}{2}\left(\frac{\partial\rho}{\partial h}\right)^2 - \rho\frac{\partial^2\rho}{\partial h^2}\right\} + \rho_2\int_h^\infty \rho\,\psi_2(u)\,du - \frac{1}{2}\rho^2\int_h^\infty \psi(u)\,du - \int_h^\infty \rho^2 \ldots$$
$$+\frac{1}{2}\int\rho\,d\rho\,\frac{d\int_h^\infty u\,\psi(u)\,du}{dh} - \frac{1}{2}\rho^2\,\frac{d\int_h^\infty u\,\psi(u)\,du}{dh} + \frac{1}{2}\int d\rho\,\frac{d^2\rho}{dh^2}\int_h^\infty u^2\psi(u)\,du$$
$$-\frac{1}{2}\rho\frac{\partial^2\rho}{\partial h^2}\int_h^\infty u^2\psi(u)\,du + \frac{1}{4}\int\rho\,d\rho\,\frac{d^2\int_h^\infty u^2\psi(u)\,du}{dh^2} - \frac{1}{4}\rho^2\,\frac{d^2\int_h^\infty u^2\psi(u)\,du}{dh^2}$$
$$+\frac{1}{2}\int\left(\frac{\partial\rho}{\partial h}\right)^2\frac{d\int_h^\infty u^2\psi(u)\,du}{dh} - \frac{1}{2}\rho\frac{\partial\rho}{\partial h}\,\frac{d\int_h^\infty u^2\psi(u)\,du}{dh}$$

h groot

$$L = -p_1$$
$$-p_1 - \rho_2\int_h^\infty \rho\,\psi_2(u)\,du = -p_\rho - c_2\left\{\frac{1}{2}\left(\frac{\partial\rho}{\partial h}\right)^2 - \rho\frac{\partial^2\rho}{\partial h^2}\right\} - \rho^2\int_h^\infty \psi(u)\,du + \int\rho\,d\rho\int_h^\infty \psi(u)\,du$$
$$-\frac{1}{2}\rho^2\,\frac{d\int_h^\infty u\,\psi(u)\,du}{dh} + \frac{1}{2}\int\rho\,d\rho\,\frac{d\int_h^\infty u\,\psi(u)\,du}{dh}$$



$\frac{dp}{dx_2} \gtrless 0$ of

$\frac{v_2 - v_1}{x_2 - x_1} \gtrless \left(\frac{dv_2}{dx_2}\right)_p$

van B tot C

$\frac{v_2 - v_1}{x_2 - x_1} < \left(\frac{dv_1}{dx_1}\right)_p$

van C tot D

$\frac{v_2 - v_1}{x_2 - x_1} > \left(\frac{dv_1}{dx_1}\right)_p$

$$\left(\frac{dv}{dx}\right)_p = \frac{\frac{MRT}{(v-b)^2}\frac{db}{dx} - \frac{da}{dx}\frac{1}{v^2}}{\frac{MRT}{(v-b)^2} - \frac{2a}{v^3}}$$

$dv = \left(\frac{\partial v}{\partial x}\right)_p dx + \left(\frac{\partial v}{\partial p}\right)_x dp$ in D $\frac{dv}{dx} = \left(\frac{\partial v}{\partial x}\right)_p$

$\frac{dv}{dx} = \left(\frac{\partial v}{\partial x}\right)_p + \left(\frac{\partial v}{\partial p}\right)_x \frac{dp}{dx}$

als $\frac{dp}{dx} > 0$

dan $\frac{dv}{dx} < \left(\frac{\partial v}{\partial x}\right)_p$

$\left(\frac{\partial v}{\partial x}\right)_p > \frac{dv}{dx}$

$p = \frac{RT(1+x)}{v-b} - \frac{a}{v^2}$ $\qquad \frac{x^2}{(1-x)(v-b)} = \varphi(T)$

$\left(\frac{\partial p}{\partial v}\right)_T = -\frac{RT(1+x)}{(v-b)^2} + \frac{RT}{v-b}\left(\frac{dx}{dv}\right)_T + \frac{2a}{v^3}$ $\qquad \left(\frac{2dx}{x} + \frac{dx}{1-x} = \frac{dv}{v-b}\right.$

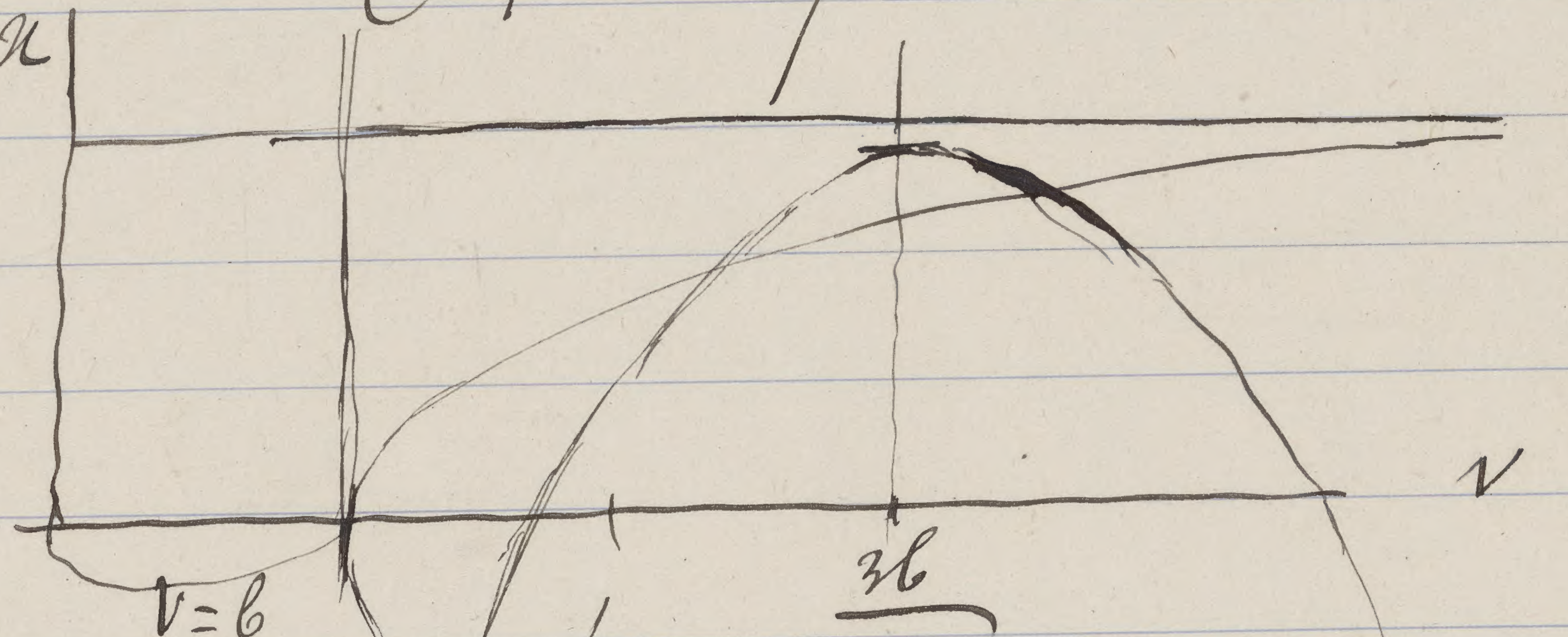
$\frac{dx}{dv}\left\{\frac{2-x}{x(1-x)}\right\} = \frac{1}{v-b}$

$\left(\frac{\partial p}{\partial v}\right)_T = -\frac{RT}{(v-b)^2}\frac{[(2-x)(1+x) - x(1-x)]}{2-x} + \frac{2a}{v^3}$

$\left(\frac{\partial p}{\partial v}\right)_T = -\frac{RT}{(v-b)^2}\frac{2-~~\dots~~^2}{2-x} + \frac{2a}{v^3}$

hoeveel male zal $\left(\frac{\partial p}{\partial v}\right)_T = 0$ zijn?

Grafisch



lijn $2 - x = \frac{RT}{a}\frac{v^3}{(v-b)^2}$ geeft ~~$\frac{dx}{dv}$~~

$x = 2 - \frac{RT}{a}\frac{v^3}{(v-b)^2}$ geeft $\frac{dx}{dv} = -\frac{RT}{a}\left\{\frac{3v^2}{(v-b)^2} - \frac{2v^3}{(v-b)^3}\right\}$

of $\left(\frac{dx}{dv}\right)_{\frac{\partial p}{\partial v}=0} = -\frac{RT}{a}\frac{v^2(v-3b)}{(v-b)^3}$ en $\left(\frac{\partial^2 x}{\partial v^2}\right) = -\frac{RT}{a}\frac{3v}{(v-b)^3}\frac{2b^2}{v-b}$

Maximum $x = 2 - \frac{RT}{a}\frac{27b}{4}$

Voor de kritt T vallende worteln samen

$$RT_k = \frac{a}{v_k^2}\frac{(v_k-b)^2}{2b}\left\{1 \pm \sqrt{\left(1 - \frac{8}{3}\frac{b}{v_k}\right)}\right\}$$

$v_k = 3b$

of bij 1 v_k twee RT_k (Evenals bij $RT_{k_{x=1}}$ & $x=0$

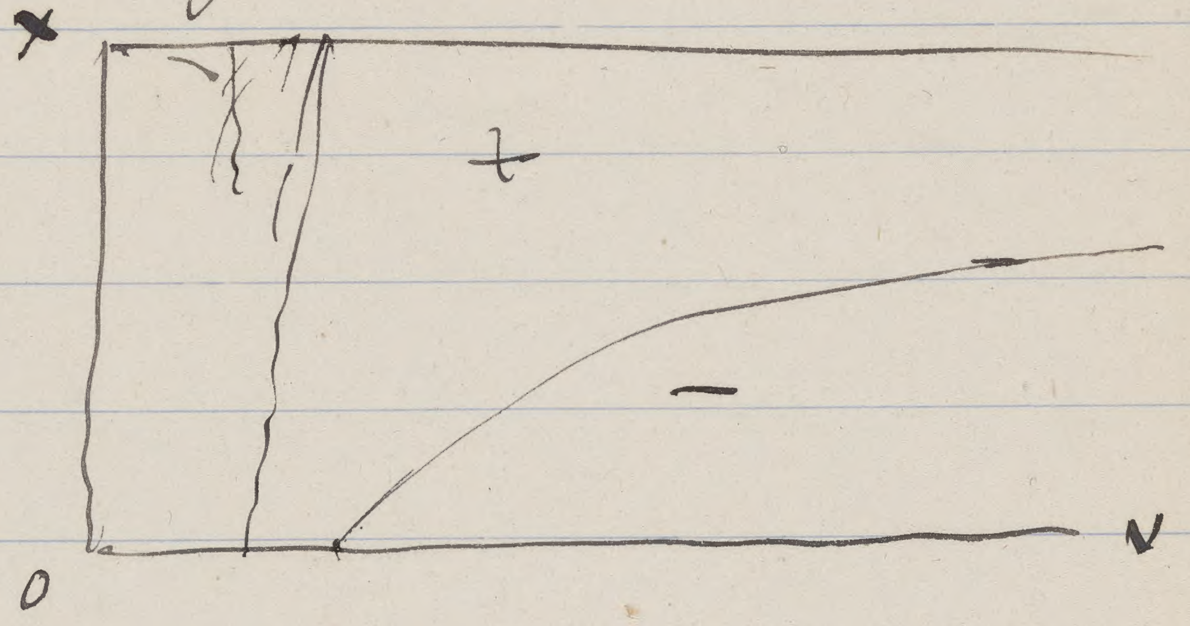
$v_k = \frac{3b}{1 + \frac{x(1-x)}{(2-x)^2}}$ Dus kleinste $v_k = \frac{8}{3}b$

$\left(\frac{dv}{dx}\right)_p = \frac{\frac{RT}{(v-b)^2}\frac{db}{dx} - \frac{da}{dx}\frac{1}{v^2}}{\frac{RT}{(v-b)^2} - \frac{2a}{v^3}}$ / $\frac{dv}{dx} > 0$ als $RT\frac{db}{dx} > \frac{da}{dx}\left(\frac{v-b}{v}\right)^2$

I $\frac{db}{dx} > 0$ en $\left(\frac{da}{dx}\right) > 0$

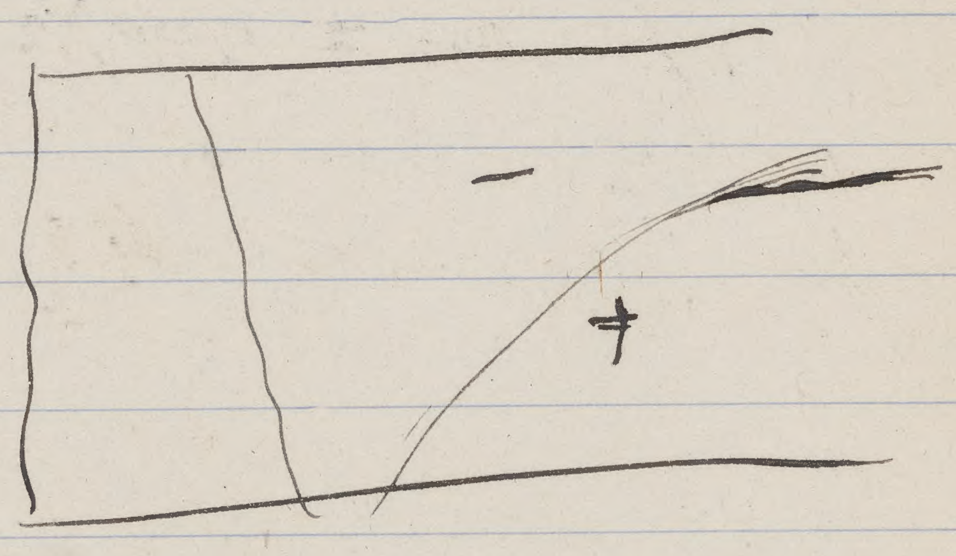
$\sqrt{\frac{RT\frac{db}{dx}}{\frac{da}{dx}}} > \frac{v-b}{v}$ of $\frac{b}{v} > 1 - \sqrt{\frac{RT\frac{db}{dx}}{\frac{da}{dx}}}$

of $v < \frac{1}{1-\sqrt{RT\frac{\frac{db}{dx}}{\frac{da}{dx}}}}$



II $\frac{db}{dx} < 0$ & $\frac{da}{dx} < 0$

$v > \frac{b}{1-\sqrt{RT\frac{\frac{db}{dx}}{\frac{da}{dx}}}}$



III

Zij CO_2 en ClH

$b_1 = 0{,}002$ $b_2 = 0{,}00175$

$a_1 = 0{,}0075$ $a_2 = 0{,}007$

Zij $a_{12} = 0{,}0072$ $RT_k = 1{,}11$

$x = 1/10$	1.052
$x = 2/10$	1.073
$x = 3/10$	1.09
$x = 4/10$	1,112
$x = 5/10$	1.134

$b_2 - b_1 = -0{,}00025$

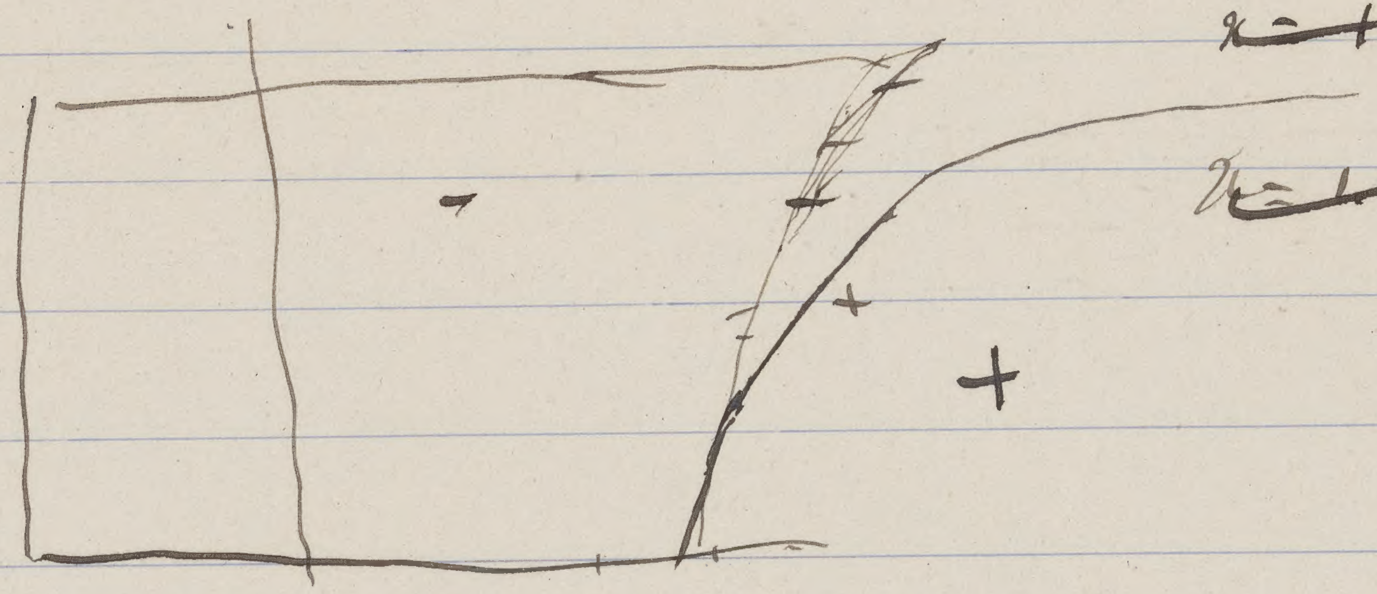
$\frac{da}{dx} = 2\{-0{,}0004 + 0{,}0008x\}$

$v > \frac{b}{1-\sqrt{\frac{RT}{2}\frac{2.5}{4-3x}}}$

$x = 0$	$v > 2.5b$
$x = 0{,}1$	
$x = \frac{1}{3}$	$v > 3.3b$
$x = 2/3$	$v > 5b$
$x = 0{,}866$	$v > \infty$

~~x = 1 v >~~

~~x = 1 v = ∞~~



Das nulldruckgebied van $\left(\frac{dv}{dx}\right) < 0$ en [illegible]

of $a_{12} = 0{,}0072$ $a = 7{,}5 - 0{,}6x + 0{,}1x^2$

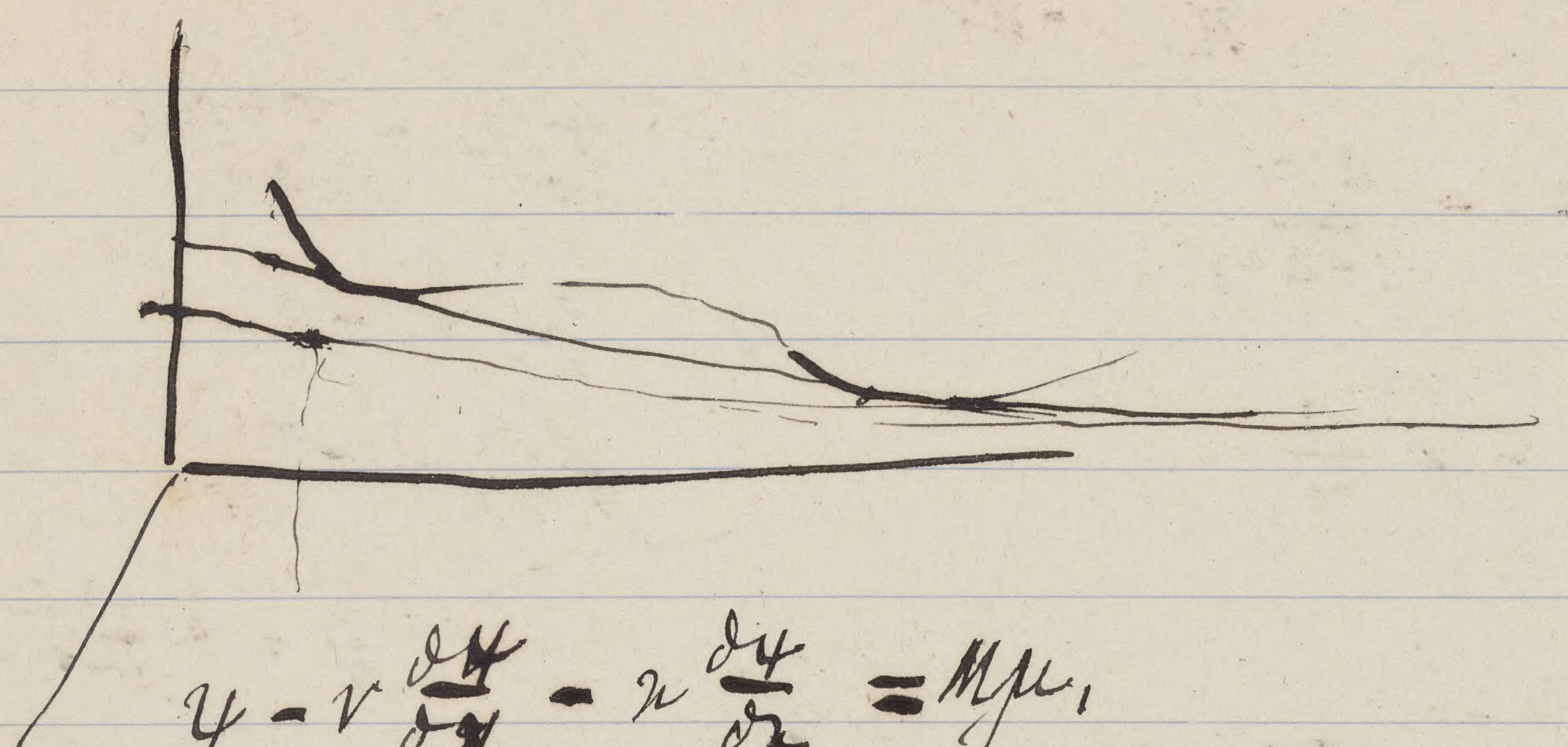
x	a		b	a/b
0	a_0 0,0075	59	200	3.75
1/10	$a_{1/10}$ 0,007441	57	1975	3.768
2	a 7384	55	195	3.787
3	7329	53	1925	3.807
4	7276	51	19	3830
5	7225	49	1875	3.851
6	7176	47	1850	
7	7129	45	1825	
8	7084	43	1800	
9	7041	41	1775	
10	7		175	

$x = 0{,}5$

$1.14V'^3 - 14{,}45\{V' - 1{,}875\}^2 = \frac{\frac{0{,}25\times4}{14{,}45}\{7.225\times0{,}25 - V'\times0{,}28\}^2}{1 - \frac{1}{1{,}14\times14{,}45}\frac{0{,}66}{V'}}$

wortel [illegible] 5,4 en 5,8 dichtbij 5,4

en Curve 5.85



$$\psi - v\frac{d\psi}{dv} - x\frac{d\psi}{dx} = M\mu_1$$

$$-MRT\log(v-b) - \frac{a}{v} + MRT\{x\log x + (1-x)\log(1-x)\}$$

$$-x\left\{MRT\frac{\frac{db}{dx}}{v-b} - \frac{da}{dx}\frac{1}{v} + MRT\log\frac{x}{1-x}\right\} + pv = M\mu_1$$

$$M_1\mu_1 = -MRT\log(v-b) + MRT\log(1-x)$$

$$-x\,MRT\frac{\frac{db}{dx}}{v-b} - \frac{1}{v}\left\{a - x\frac{da}{dx}\right\} + pv$$

in gastoestand

$$M_1\mu_1 = -MRT\log\frac{MRT}{p}$$

~~$\Delta M_1\mu_1 = -MRT\frac{dp}{p}$~~

of $-MRT\log(v_1-b_1) - \frac{a_1}{v_1} + p_0 v_1 = MRT\log\frac{p_0}{MRT}$

$$-MRT\log(v-b_x) - \frac{\left(a_x - x\frac{da_x}{dx}\right)}{v} + pv = MRT\log\frac{p}{MRT}$$

$$+MRT\log(1-x) \quad - x\frac{MRT\frac{db}{dx}}{v-b}$$

~~$MRT\frac{\frac{db}{dx}}{v-b} - \frac{da}{dx}\frac{1}{v} + MRT$~~

$x_1 = \Delta x$
$x_2 = 0$

$$\left\{v_2 - v_1 - (x_2 - x_1)\frac{dv_1}{dx_1}\right\}dp = (x_2 - x_1)dx_1\left\{\frac{MRT}{x_1(1-x_1)}\right. \text{ enz}$$

$$(v_2 - v_1)\frac{dp}{dx_1} = -\frac{MRT}{(1-x)} \quad / \quad \frac{dp}{dx_1} = -\frac{MRT}{v}$$

Dus $\frac{MRT}{p}\Delta p = \frac{-x_1\,\Delta x_1}{x_1(1-x_1)}MRT$

$$\frac{MRT}{p}\Delta p = -\frac{\Delta x_1}{1-\Delta x_1}$$

of $\frac{p_0 - p}{p_0} = +\frac{n}{n_0}$

$$\frac{T}{p}\frac{dp}{dT} = \frac{r+\lambda}{RT}p$$

en $\frac{dp}{dn} = -p$

$$\frac{dn}{dT} = -\frac{r+\lambda}{RT^2}$$

of $\frac{n}{n_0} = \frac{(r+\lambda)M(T_0 - T_1)}{MRT^2}$

$$\frac{MRT}{p}\frac{\Delta p}{\Delta x_1} = \frac{x_2 - x_1}{x_1}\frac{1}{1-x_1} \quad \text{bij kleine } x$$

606.5
79.5
~~605.5×10~~ 1.2
2.3 ... $\frac{10334}{...}$

kleinste waarde als $x_2 = 0$
$= 0$ als $x_2 = x_1$
$+$ als $x_2 > x_1$

$$\psi = -MRT\log(v-b) - \frac{a}{v} + MRT[x\,l\,x + (1-x)\,l(1-x)]$$

$$\frac{d\psi}{dx} = MRT\frac{\frac{db}{dx}}{v-b} - \frac{da}{dx}\frac{1}{v} + MRT\log\frac{x}{1-x}$$

$$\frac{d\psi}{dx} = M_2\mu_2 - M_1\mu_1$$

$$MRT\frac{\frac{db}{dx}}{v_1-b} - \frac{da}{dx}\frac{1}{v_1} + MRT\log\frac{x_1}{1-x_1} = \frac{MRT}{v_2-b}\frac{db}{dx} - \frac{da}{dx}\frac{1}{v_2} + MRT\log\frac{x_2}{1-x_2}$$

$$MRT\log\frac{x_1}{x_2}\frac{1-x_2}{1-x_1} = \left(p + \frac{a}{v_2^2}\right)\frac{db}{dx} - \frac{da}{dx}\frac{1}{v_2} - \left(p + \frac{a}{v_1^2}\right)\frac{db}{dx} + \frac{da}{dx}\frac{1}{v_1}$$

$$MRT\log\frac{x_1}{x_2}\frac{1-x_2}{1-x_1} = \left(\frac{1}{v_1} - \frac{1}{v_2}\right)\frac{da}{dx} - \left(\frac{a}{v_1^2} - \frac{a}{v_2^2}\right)\frac{db}{dx}$$

$$MRT\log\frac{x_1}{x_2}\frac{1-x_2}{1-x_1} = \left(\frac{1}{v_1} - \frac{1}{v_2}\right)\left\{\frac{da}{dx} - a\frac{db}{dx}\left(\frac{1}{v_1} + \frac{1}{v_2}\right)\right\}$$

bij benadering

$$MRT\log\frac{x_1}{x_2}\frac{1-x_2}{1-x_1} = \frac{v_2 - v_1}{v_1 v_2}\left\{\frac{da}{dx} - a\frac{db}{dx}\frac{2}{v}\right\}\frac{1}{v_1} = \frac{1}{v_1}\left\{\frac{da}{dx} - \frac{a}{v_1}\frac{db}{dx}\right\}$$

$$= \frac{1}{v_1}\left\{2(a_{12} - a_1) - \frac{a_1}{v_1}(b_2 - b_1)\right\}$$

2de geval $\frac{x_2}{x_1} = \infty$ (omgekeerd)

$$v_2 \frac{dp}{dx_2} = (x_2 - x_1) \frac{MRT}{x_2(1-x_2)}$$

bijv. water gedeeltelijk ingeperst wordt

$$\frac{MRT}{p} \frac{dp}{dx_2} = \frac{1}{1-x_2}$$

of $\frac{p - p_0}{p_0} = \frac{n}{n_0}$

of $\frac{\text{druk lucht}}{\text{druk waterdamp}} = \frac{n_e}{n_w}$

of door $\frac{\partial^2\psi}{\partial x_2^2} - \frac{\left(\frac{\partial^2\psi}{\partial x \partial v}\right)^2}{\frac{\partial^2\psi}{\partial v^2}}$ ~~te~~ verdampbaarheid $\frac{MRT}{x_2(1-x_2)}$

$$\frac{MRT}{p} \frac{dp}{dx_2} = x_2 \frac{MRT\, dx_2}{x_2(1-x_2)}$$

$$lp = l \frac{c}{1-x_2}$$

$$p(1-x_2) = C = p_{x_2=0}$$

$$p = \frac{p_w}{1-x_2} = p_w + p_w \frac{x}{1-x_2}$$

$$p - p_w = p_w \frac{x_2}{1-x_2}$$

of $\frac{p_e}{p_w} = \frac{x}{1-x}$

Uitgaande ~~Uitgaande~~ ~~de~~ van zoutwater & waterdamp zal overgegaan worden tot het geval van water + zout en damp

is $v_1 dp = d\, M_1\mu_1 + x_1 d(M_2\mu_2 - \mu_1)$

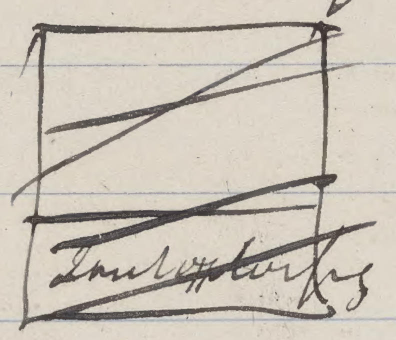
$V_2 dp = d\, M_1\mu_1 + 0 . d(M_2\mu_2 - \mu_1)$

of $d(M_1\mu_1) = V_2 dp$ (waarin $dp = -p\,dx_1$)

~~of de vermeerdering~~

De vermeerdering van potentiaal als een molekuul water zich in zuiver water bevindt, of dat het zich bevindt in de zoutoplossing is $d(M_1\mu_1) = V_2 dp$

~~[illegible] afleiden~~ ..



~~$M_1\mu_1 =$~~ Daar dp negatief is, is de vermindering van arbeidsvermogen van plaats

Stellen wij twee deelen van een membraan waarin water; ~~[illegible]~~ gedeelte ~~is de potentiaal~~ P is de potentiaal van uitwendige krachten C, terwijl overal $C+P$

dan wordt $\epsilon' - T\eta' + p'V_1 + C + P = M_1\mu_1$

of $V_1 dp' + dP$

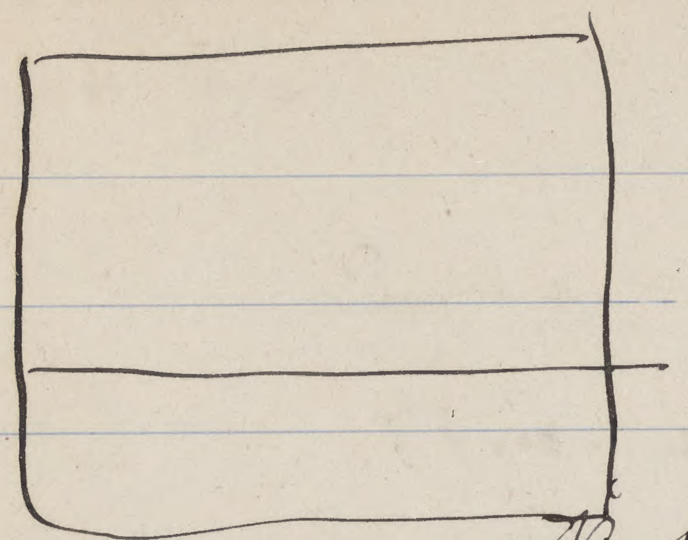
$V_1 dp' = -V_2 dp$

of $\frac{\text{osmdruk}}{\text{drukverlaging}} = \frac{V_2}{V_1}$

---

daar $v_1 dp' = +p V_2 dx_1$

$$dp' = \frac{MRT}{V_1} dx_1$$



In het onderste gedeelte
ether en etherdamp
boven lucht (etherdamp kan met deze
grenzende [?].

In lucht overal gelijke $M_1 \mu_1$

In het mengsel $\psi + pv - x\frac{d\psi}{dx} = MRT + MRT\log\frac{p'(1-x)}{MRT}$

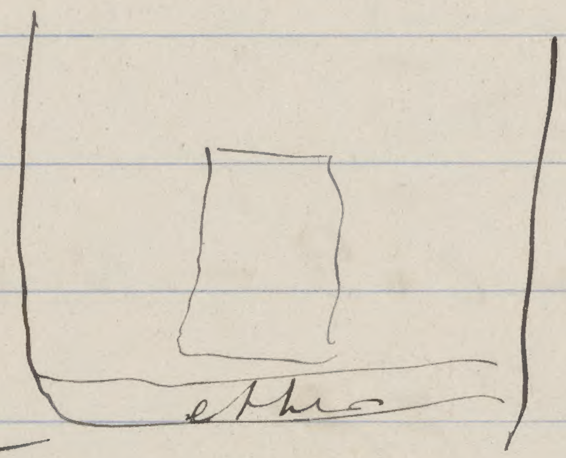
boven $MRT + MRT\log\frac{p^*}{MRT}$

of $p = p'(1-x)$

$p' = \frac{p}{1-x} = p + p\frac{x}{1-x}$

$\frac{p'-p}{p'} \equiv \frac{x}{1-x}$

Staat het onderste in gemeenschap met de atmosfeer en het bovenste afgesloten



Dus bij eenzijdige toename in druk
bij evenwicht [?] (verminderd arbeidsv. in plaats)

DSI

~~Thermodynamische~~
Meer syst. behandeling bij samenhangende
ε' (thermo energie) [?] Potentiaal

~~$\psi = \varepsilon' - T\eta + p$~~ $\psi = \psi' + (1-x)M_1P_1 + xM_2P_2$

waarin P_1 en P_2 functies van $\xi\eta\zeta$

$\int \frac{\psi}{v} d\xi d\eta d\zeta =$ min. $\int \frac{M_1(1-x)}{v} d\xi d\eta d\zeta = C_1$ en $\int \frac{M_2 x}{v} d\xi d\eta d\zeta = C_2$

of $\int \frac{\psi' - M_1(1-x)(\mu_1 - P_1) - xM_2(\mu_2 - P_2)}{v} d\xi d\eta d\zeta$ een minimum

Dus (I) $\frac{\partial\left[\frac{\psi' - M_1(1-x)(\mu_1-P_1) - M_2x(\mu_2-P_2)}{v}\right]}{\partial v_{x\xi\eta\zeta}} = 0$ en (II) $\frac{\partial\left[\frac{\psi' - M_1(1-x)(\mu_1-P_1) - M_2x(\mu_2-P_2)}{v}\right]}{\partial x_{v\xi\eta\zeta}} = 0$

uit I $-\frac{[\psi' - M_1(1-x)(\mu_1-P_1) - M_2x(\mu_2-P_2)]}{v} + \frac{d\psi'}{dv_x} = 0$ of $\frac{d\psi'}{dv_x} = -p = -f(\xi\eta\zeta)$

uit II $\frac{d\psi'}{dx_v} = M_2(\mu_2-P_2) - M_1(\mu_1-P_1)$

~~Stel opgemerkt $\psi' = F(x, v)$ geconstrueerd~~

~~De verschillende stukken~~ Zij het opgemerkte $\psi' = f(x,v)$
(A) ~~Zijn er geen uitwendige krachten~~ geconstrueerd, dan in elk punt een
waarde aangebracht, dan is door $\left(\frac{d\psi'}{dv}\right)_x = -p$ en $\left(\frac{d\psi'}{dx}\right)_v = M_2(\mu_2-P_2) - M_1(\mu_1-P_1)$
de richting van het [?] gegeven, de stukken der assen afsnijden
zijn $\psi' - v\frac{d\psi'}{dv} - x\frac{d\psi}{dx} = M_1(\mu_1-P_1)$ en $M_2(\mu_2-P_2)$ — dan
zal (A) als P_1 en P_2 = ~~constanten~~ 0 zijn ~~zoowel~~ voor
alle phasen die in het vat kunnen voorkomen de stukken
der assen afsnijden en $\frac{d\psi'}{dx}$ en dus ook $\frac{d\psi'}{dv}$ ~~bepaald~~ maken
zijn [?] dus eenzelfde gemeenschappelijk raakvlak aan al die
phasen kunnen gebracht worden.

B als P_1 en P_2 waarde hebben dan moeten al de stukken der assen
A_1 en A_2 zijn. $A_1 + M_1P_1$ standvastig zijn en $A_2 + M_2P_2$ standvastig

Dus als P_1 en P_2 waarde hebben

is I $\left(\frac{\partial\psi'}{\partial x}\right)_v = M_1 P_1 - M_2 P_2$

en II $\psi' - x\frac{\partial\psi'}{\partial x} + pv + M_1 P_1 =$ Constant

of ~~$Vdp +$~~

$$d\psi' - x\, d\frac{\partial\psi'}{\partial x} - \frac{\partial\psi'}{\partial x}dx + v\,dp + p\,dv + M_1 dP_1 = 0$$

$$d\psi' - \frac{\partial\psi'}{\partial x}dx + p\,dv = 0$$

of $V dp + M_1(1-x)\,dP_1 + M_2 x\, dP_2 = 0$

~~$Vdp +$~~

uit I $\left\{\frac{\partial^2\psi'}{\partial x^2}dx_v + \frac{\partial^2\psi'}{\partial x\partial v}dv\right\} = M_1 dP_1 - M_2 dP_2$

of $\frac{\partial^2\psi'}{\partial x^2}dx + \frac{\partial^2\psi'}{\partial x\partial v}\left[\left(\frac{\partial v}{\partial x}\right)_p dx + \left(\frac{\partial v}{\partial p}\right)_p dp\right] = M_1 dP_1 - M_2 dP_2$

$$\left\{\frac{\partial^2\psi'}{\partial x^2} - \frac{\left(\frac{\partial^2\psi'}{\partial x\partial v}\right)^2}{\frac{\partial^2\psi}{\partial v^2}}\right\}dx \mp \left(\frac{\partial v}{\partial x}\right)_p dp = M_1 dP_1 - M_2 dP_2$$

$$\left\{\frac{\partial^2\psi'}{\partial x^2} - \frac{\left(\frac{\partial^2\psi'}{\partial x\partial v}\right)^2}{\frac{\partial^2\psi}{\partial v^2}}\right\}dx \mp \frac{1}{V}\left(\frac{\partial v}{\partial x}\right)_p\left\{M_1(1-x)\,dP_1 + M_2 x\, dP_2\right\} = M_1 dP_1 - M_2 dP_2$$

$$\left\{\frac{\partial^2\psi'}{\partial x^2} - \frac{\left(\frac{\partial^2\psi}{\partial x\partial v}\right)^2}{\frac{\partial^2\psi}{\partial v^2}}\right\}dx = dP_1 M_1\left\{\frac{V + (1-x)\left(\frac{\partial v}{\partial x}\right)_p}{V}\right\} - dP_2 M_2\left\{\frac{V \mp x\left(\frac{\partial v}{\partial x}\right)_p}{V}\right\}$$

is $P_1 = P_2 = gh$

$$\text{dan } \left(\frac{\partial^2\psi'}{\partial x^2} - \frac{\left(\frac{\partial^2\psi}{\partial x\partial v}\right)^2}{\frac{\partial^2\psi}{\partial v^2}}\right)dx = g\,dh\left\{\frac{(M_1 - M_2)V + \left(\frac{\partial v}{\partial x}\right)_p\left[M_1(1-x) + M_2 x\right]}{V}\right\}$$

~~$\rho = \frac{M_1(1-x) + M_2 x}{V}$~~

~~$\left(\frac{d\rho}{dx}\right)_p = \frac{V(M_1 - M_2) + \left[M_1(1-x) + M_2 x\right]\left(\frac{dv}{dx}\right)_p}{V^2}$~~

~~$\left\{\frac{\partial^2\psi'}{\partial x^2} - \frac{\left(\frac{\partial^2\psi}{\partial x\partial v}\right)^2}{\left(\frac{\partial^2\psi}{\partial v^2}\right)}\right\}dx = g\,dh\,\frac{1}{\rho}\left(\frac{d\rho}{dx}\right)_p$~~ n.t. als x het aantal molekulen voorstelt

~~Intusschen~~

~~$\left\{\frac{\partial^2\psi'}{\partial x^2} - \frac{\left(\frac{\partial^2\psi}{\partial x\partial v}\right)^2}{\frac{\partial^2\psi}{\partial v^2}}\right\}dx = -dP_1 V\frac{\partial\left[\frac{M_1(1-x)}{V}\right]}{\partial x_p} - dP_2 V\frac{\partial\left(\frac{M_2 x}{V}\right)}{\partial x_p}$~~

~~$= -dP_1\frac{1}{\rho}\left(\frac{d\rho_1}{dx}\right)_p - dP_2\frac{1}{\rho}\left(\frac{d\rho_x}{dx}\right)_p$~~

~~N.B. als x het aantal molekulen voorstelt~~

Voorwegens

$$\frac{MRT}{x(1-x)}dx = g\,dh\,(M_1 - M_2)$$

~~bij evenwicht~~

~~$-\frac{1}{\rho}\frac{d\rho}{dx} = \frac{-M_2 b_1 + b_2 M_1}{\left[M_1(1-x) + M_2 x\right]\left[b_1(1-x) + b_2 x\right]}$~~

~~$\left\{\frac{\partial^2\psi'}{\partial x^2} - (\quad)\right\}\frac{dx}{dh} = g\frac{b_2 M_1 - b_1 M_2}{\left[M_1(1-x) + M_2 x\right]\left[b_1(1-x) + b_2 x\right]}$~~

bij evenwichtstoestand

$$(\quad)\frac{dx}{dh} = g\,\frac{M_1 b_2 - M_2 b_1}{b_1(1-x) + b_2 x} = g\,\frac{b_1 b_2}{b_1(1-x) + b_2 x}\left(\frac{M_1}{b_1} - \frac{M_2}{b_2}\right)$$

$$\left(\frac{MRT}{x(1-x)} - \frac{\partial^2 a}{\partial x^2}\frac{1}{V}\right)\left(\frac{MRT}{(V-b)^2} - \frac{2a}{V^3}\right) - \frac{2a}{V^2}\frac{MRT}{(V-b)^2}\left(\frac{db}{dx}\right)^2 + 2\frac{da}{dx}\frac{1}{V^2}\frac{MRT}{(V-b)^2}\frac{db}{dx} - \left(\frac{da}{dx}\right)^2\frac{1}{V^4} = 0$$

$$p = 0 \qquad \frac{MRT}{V-b} = \frac{a}{V^2}$$

$$V^2 = \frac{a}{MRT}(V-b)$$

$$(V-b)^2 = \frac{a}{MRT}(V-b) - 2bV + b^2$$

$$(V-b)^2 = \left(\frac{a}{MRT} - 2b\right)V + b\left(\frac{a}{MRT} - b\right)$$

$$\left(\frac{MRT\,V}{x(1-x)} - \frac{\partial^2 a}{\partial x^2}\right)\left[MRT\,V^3 - 2a(V-b)^2\right] - 2aV\,MRT\left(\frac{db}{dx}\right)^2 + 2V^2\frac{da}{dx}MRT\frac{db}{dx} - \left(\frac{da}{dx}\right)^2(V-b)^2 = 0$$

$$V^3 = \frac{a}{MRT}\left\{\frac{a}{MRT}(V-b) - bV\right\}$$

$$V^3 = \frac{a}{MRT}\left\{\left(\frac{a}{MRT} - b\right)V - \frac{ab}{MRT}\right\}$$

$$\frac{(MRT)^2}{x(1-x)}V^4 - \frac{\partial^2 a}{\partial x^2}MRT\,V^3$$

$$\left(\frac{MRT\,V}{x(1-x)} - \frac{\partial^2 a}{\partial x^2}\right)\left\{MRT\,V^3 - 2a(MRT)^2\frac{V^4}{a^2}\right\} - 2aV\,MRT\left(\frac{db}{dx}\right)^2 + 2V^2\frac{da\,db}{dx\,dx}MRT - (MRT)\left(\frac{da}{dx}\right)^2\frac{V^4}{a^2} = 0$$

$$\left(\frac{MRT\,V}{x(1-x)} - \frac{\partial^2 a}{\partial x^2}\right)\left\{V^2 - 2\,MRT\frac{V^3}{a}\right\} - 2a\left(\frac{db}{dx}\right)^2 + 2V\frac{da\,db}{dx\,dx} - MRT\frac{\left(\frac{da}{dx}\right)^2}{a^2}V^3 = 0$$

$$-\frac{2}{a}\frac{(MRT)^2}{x(1-x)}V^4 + \left(2MRT\frac{\frac{\partial^2 a}{\partial x^2}}{a} + \frac{MRT}{x(1-x)} - MRT\frac{\left(\frac{da}{dx}\right)^2}{a^2}\right)V^3 - \frac{\partial^2 a}{\partial x^2}V^2 + 2V\frac{da}{dx}\frac{db}{dx} - 2a\left(\frac{db}{dx}\right)^2 = 0$$

$$V^2 = \frac{a}{MRT}V - \frac{ab}{MRT}$$

$$\left(V - \frac{a}{2MRT}\right)^2 = \frac{a^2}{4(MRT)^2} - \frac{ab}{MRT}$$

$$MRT\,V^3 = a\left\{\frac{a}{MRT}V - \frac{ab}{MRT} - bV\right\} = a\left\{\left(\frac{a}{MRT} - b\right)V - \frac{ab}{MRT}\right\}$$

$$(MRT)^2V^4 = a^2\left\{\left(\frac{a}{MRT} - 2b\right)V - b\left(\frac{a}{MRT} - b\right)\right\}$$

$$-\frac{2a}{x(1-x)}\left\{\left(\frac{a}{MRT} - 2b\right)V - b\left(\frac{a}{MRT} - b\right)\right\} + \left\{\frac{2\frac{\partial^2 a}{\partial x^2}}{a} + \frac{1}{x(1-x)} - \frac{\left(\frac{da}{dx}\right)^2}{a^2}\right\}a\left\{\left(\frac{a}{MRT} - b\right)V - \frac{ab}{MRT}\right\} - \left(\frac{\partial^2 a}{\partial x^2}\right)\left\{\frac{a}{MRT}V - \frac{ab}{MRT}\right\} + 2V\frac{da}{dx}\frac{db}{dx} - 2a\left(\frac{db}{dx}\right)^2 = 0$$

$$V\left\{-\frac{2a}{x(1-x)}\left(\frac{a}{MRT} - 2b\right) + \left(\frac{a}{MRT} - b\right)\left(2\frac{\partial^2 a}{\partial x^2} - \frac{\left(\frac{da}{dx}\right)^2}{a} + \frac{a}{x(1-x)}\right) - \frac{a}{MRT}\frac{\partial^2 a}{\partial x^2} + 2\frac{da}{dx}\frac{db}{dx}\right\} + \left\{\frac{2ab}{x(1-x)}\left(\frac{a}{MRT} - b\right) - \frac{ab}{MRT}\left(2\frac{\partial^2 a}{\partial x^2} - \frac{\left(\frac{da}{dx}\right)^2}{a} + \frac{a}{x(1-x)}\right) + \frac{ab}{MRT}\frac{\partial^2 a}{\partial x^2} - 2a\left(\frac{db}{dx}\right)^2\right\} = 0$$

$$\left(V - \frac{a}{2MRT}\right)\left\{-\frac{2a}{x(1-x)}\left(\frac{a}{MRT} - 2b\right) + \left(\frac{a}{MRT} - b\right)\left(2\frac{\partial^2 a}{\partial x^2} - \frac{\left(\frac{da}{dx}\right)^2}{a} + \frac{a}{x(1-x)}\right) - \frac{a}{MRT}\frac{\partial^2 a}{\partial x^2} + 2\frac{da}{dx}\frac{db}{dx}\right\} + \left\{\frac{2ab\left(\frac{a}{MRT} - b\right) - \frac{a^2}{MRT}\left(\frac{a}{MRT} - 2b\right) + \frac{a^2}{2MRT}\left(\frac{a}{MRT} - b\right) - \frac{a^2 b}{MRT}}{x(1-x)} + \left(2\frac{\partial^2 a}{\partial x^2} - \frac{\left(\frac{da}{dx}\right)^2}{a}\right)\left(-\frac{ab}{MRT} + \frac{a}{2MRT}\left(\frac{a}{MRT} - b\right)\right) + \frac{\partial^2 a}{\partial x^2}\left(\frac{ab}{MRT} - \frac{a^2}{2(MRT)^2}\right) - 2a\left(\frac{db}{dx}\right)^2 + \frac{a}{MRT}\frac{da}{dx}\frac{db}{dx}\right\}$$

$$\left(V - \frac{a}{2MRT}\right)\left\{-\frac{a\left(\frac{a}{MRT}-3b\right)}{x(1-x)} + \frac{\partial^2 a}{\partial x^2}\left(\frac{a}{MRT}-2b\right) - \frac{\left(\frac{da}{dx}\right)^2}{a}\left(\frac{a}{MRT}-b\right) + 2\frac{da}{dx}\frac{db}{dx}\right\}^2\left\{\frac{a^2}{4MRT^2} - \frac{ab}{MRT}\right\} * =$$

$$+\left\{-\frac{a\left(\frac{a}{MRT}-4b\right)\left(\frac{a}{MRT}-b\right)}{2x(1-x)} + \frac{a}{2MRT}\frac{\partial^2 a}{\partial x^2}\left(\frac{a}{MRT}-4b\right) - \left(\frac{da}{dx}\right)^2\frac{\left(\frac{a}{MRT}-3b\right)}{2MRT} + \frac{a}{MRT}\frac{da}{dx}\frac{db}{dx} - 2a\left(\frac{db}{dx}\right)^2\right\}^2 \neq 0$$

of $$\left\{-\frac{a\left[\frac{a^2}{(MRT)^2} - 4\frac{ab}{MRT} + 2b^2\right]}{x(1-x)} + \frac{a}{MRT}\frac{\partial^2 a}{\partial x^2}\left(\frac{a}{MRT}-3b\right) - \left(\frac{da}{dx}\right)^2\frac{\frac{a}{MRT}-2b}{MRT} + 2\frac{a}{MRT}\frac{da}{dx}\frac{db}{dx} - 2a\left(\frac{db}{dx}\right)^2\right\} \times$$

$$\left\{\frac{ab\left(\frac{a}{MRT}-2b\right)}{x(1-x)} - \frac{a}{MRT}b\frac{\partial^2 a}{\partial x^2} + b\left(\frac{da}{dx}\right)^2\frac{1}{MRT} - 2a\left(\frac{db}{dx}\right)^2\right\} + \frac{ab}{MRT}\left\{\frac{-a\left(\frac{a}{MRT}-3b\right)}{x(1-x)} + \frac{\partial^2 a}{\partial x^2}\left(\frac{a}{MRT}-2b\right) - \frac{\left(\frac{da}{dx}\right)^2}{a}\left(\frac{a}{MRT}-b\right) + 2\frac{da}{dx}\frac{db}{dx}\right\}^2 = 0$$

of $$\left\{(MRT)^2\left[\frac{-2ab^2}{x(1-x)} - 2a\left(\frac{db}{dx}\right)^2\right] + MRT\left[\frac{4a^2b}{x(1-x)} - 3ab\frac{\partial^2 a}{\partial x^2} + 2\left(\frac{da}{dx}\right)^2 b + 2a\frac{da}{dx}\frac{db}{dx}\right] + \left[-\frac{a^3}{x(1-x)} + a^2\frac{\partial^2 a}{\partial x^2} - a\left(\frac{da}{dx}\right)^2\right]\right\} \times$$

$$\left\{MRT\left[-\frac{2ab^2}{x(1-x)} - 2a\left(\frac{db}{dx}\right)^2\right] + \left[\frac{a^2 b}{x(1-x)} - ab\frac{\partial^2 a}{\partial x^2} + b\left(\frac{da}{dx}\right)^2\right]\right\} + ab\left\{MRT\left[\frac{3ab}{x(1-x)} - 2b\frac{\partial^2 a}{\partial x^2} + \frac{b}{a}\left(\frac{da}{dx}\right)^2 + 2\frac{da}{dx}\frac{db}{dx}\right] + \left[-\frac{a^2}{x(1-x)} + a\frac{\partial^2 a}{\partial x^2} - \left(\frac{da}{dx}\right)^2\right]\right\}^2 = 0$$

Term van MRT onafhankelijk $-a\left[-\frac{a^2}{x(1-x)} + a\frac{\partial^2 a}{\partial x^2} - \left(\frac{da}{dx}\right)^2\right]^2 b + ab\left[\quad\right]^2 = 0$

Coëff van MRT is $ab^3\left[\frac{a^2}{x(1-x)} - a\frac{\partial^2 a}{\partial x^2} + \left(\frac{da}{dx}\right)^2\right]\left[\frac{1}{b}\frac{\partial^2 a}{\partial x^2} - \frac{2}{b^2}\frac{da}{dx}\frac{db}{dx} + \frac{2a}{b^3}\left(\frac{db}{dx}\right)^3\right] = ab^3\left[\frac{a^2}{x(1-x)} - a\frac{\partial^2 a}{\partial x^2} + \left(\frac{da}{dx}\right)^2\right]\frac{d^2\frac{a}{b}}{dx^2}$

~~$+ \ldots \frac{da}{dx}\frac{db}{dx} - 10a^2b\left(\frac{db}{dx}\right)^2$~~

~~Coëff van MRT^2 is $\frac{-a^3b^3}{x^2(1-x)^2} + \frac{-4a^2b^3\frac{\partial^2 a}{\partial x^2} + 4ab^3\left(\frac{da}{dx}\right)^2}{x(1-x)}$~~

$$\left[T^2A + TB + C\right]\left[TA - \frac{b}{a}C\right] + \frac{ab}{}\left[LT + \frac{C}{a}\right]^2 = 0 = T^2A^2 + T\left\{A\left(B - \frac{b}{a}C\right) + abL^2\right\} + C\left\{A - \frac{b}{a}B + 2bL\right\} = 0$$

$A = -2a\left\{\frac{b^2}{x(1-x)} + \left(\frac{db}{dx}\right)^2\right\}$

$-\frac{b}{a}B = -2a\left\{\frac{2b^2}{x(1-x)} - \frac{3}{2}\frac{b^2}{a}\frac{\partial^2 a}{\partial x^2} + b^2\frac{1}{a^2}\left(\frac{da}{dx}\right)^2 + \frac{b}{a}\frac{da}{dx}\frac{db}{dx}\right\}$ of $A - \frac{b}{a}B + 2bL = 2a\left\{-\frac{1}{2}\frac{b^2}{a}\frac{\partial^2 a}{\partial x^2} + \frac{b}{a}\frac{da}{dx}\frac{db}{dx} - \left(\frac{db}{dx}\right)^2\right\} = -b^3\frac{\partial^2\frac{a}{b}}{dx^2}$

$2bL = 2a\left\{\frac{3b^2}{x(1-x)} - 2\frac{b^2}{a}\frac{\partial^2 a}{\partial x^2} + \frac{b^2}{a^2}\left(\frac{da}{dx}\right)^2 + \frac{2b}{a}\frac{da}{dx}\frac{db}{dx}\right\}$

of anders

$$V\left\{T\left[\frac{3ab}{x(1-x)}-b\left(2\frac{\partial^2 a}{\partial x^2}-\frac{\left(\frac{da}{dx}\right)^2}{a}\right)+2\frac{da}{dx}\frac{db}{dx}\right]-\left[\frac{a^2}{x(1-x)}-a\left(2\frac{\partial^2 a}{\partial x^2}-\frac{\left(\frac{da}{dx}\right)^2}{a}\right)+a\frac{\partial^2 a}{\partial x^2}\right]\right\}+$$

$$+\left\{-T2a\left(\frac{b^2}{x(1-x)}+\left(\frac{db}{dx}\right)^2\right)+b\left[\frac{a^2}{x(1-x)}-a\left(2\frac{\partial^2 a}{\partial x^2}-\frac{\left(\frac{da}{dx}\right)^2}{a}\right)+a\frac{\partial^2 a}{\partial x^2}\right]\right\}$$

Zy

$$K=\frac{3ab}{x(1-x)}-b\left[2\frac{\partial^2 a}{\partial x^2}-\frac{\left(\frac{da}{dx}\right)^2}{a}\right]+2\frac{da}{dx}\frac{db}{dx}$$

$$L=-\left\{\frac{a^2}{x(1-x)}-a\left[2\frac{\partial^2 a}{\partial x^2}-\frac{\left(\frac{da}{dx}\right)^2}{a}\right]+a\frac{\partial^2 a}{\partial x^2}\right\}$$

$$M=-2a\left[\frac{b^2}{x(1-x)}+\left(\frac{db}{dx}\right)^2\right]$$

of $V=-\dfrac{MT-bL}{KT+L}=\dfrac{bL-MT}{KT+L}=b-\dfrac{(bK+M)T}{KT+L}$

$V^2=\dfrac{a}{T}(V-b)$ voor $p=0$

$$\frac{(MT-bL)^2}{(KT+L)^2}=-\frac{a}{T}\frac{(bK+M)T}{KT+L}$$

of $(MT-bL)^2=-a(bK+M)(KT+L)$

of $M^2T^2+T\{aK(bK+M)-2bML\}+\{a(bK+M)+b^2L\}L=0$

Nu is $a(bK+M)+b^2L=\dfrac{3a^2b^2-2a^2b^2-a^2b^2}{x(1-x)}-ab[\;]\ \{a^2b-a^2b\}+\dfrac{da}{dx}\dfrac{db}{dx}\tfrac{1}{b}2ab-ab^2\dfrac{\partial^2 a}{\partial x^2}-2a^2\left(\dfrac{db}{dx}\right)^2$

of $a(bK+M)+b^2L=-ab^3\left\{\dfrac{1}{b}\dfrac{\partial^2 a}{\partial x^2}-\dfrac{2}{b^2}\dfrac{da}{dx}\dfrac{db}{dx}+\dfrac{2a}{b^3}\left(\dfrac{db}{dx}\right)^2\right\}=-ab^3\dfrac{d^2\frac{a}{b}}{dx^2}$

als $\dfrac{d^2\frac{a}{b}}{dx^2}=0$ dan

of $a(bK+M)=-ab^3\dfrac{d^2\frac{a}{b}}{dx^2}-b^2L$

$$M^2T-\left\{ab^3K\frac{d^2\frac{a}{b}}{dx^2}+Kb^2L+2bML\right\}$$

$$T=\frac{Kb+2M}{M^2}Lb$$

$$T=\frac{a}{4b}\,\frac{\left\{1+x(1-x)\left(-\frac{1}{a}[\;]+\frac{1}{a}\frac{\partial^2 a}{\partial x^2}\right)\right\}\left\{1+x(1-x)\left[\frac{1}{a}[\;]-2\frac{1}{a}\frac{da}{dx}\frac{1}{b}\frac{db}{dx}+\frac{4}{b^2}\left(\frac{db}{dx}\right)^2\right]\right\}}{\left\{1+\frac{1}{b^2}\left(\frac{db}{dx}\right)^2x(1-x)\right\}^2}$$

$$T=\frac{a}{4b}\left\{1+\frac{x(1-x)\,\frac{b}{a}\frac{d^2\frac{a}{b}}{dx^2}+x^2(1-x)^2\left\{\left(-\frac{[\;]}{a}+\frac{\partial^2 a}{\partial x^2}\right)\left(\frac{[\;]}{a}-2\frac{1}{a}\frac{da}{dx}\frac{db}{dx}+\frac{4}{b^2}\left(\frac{db}{dx}\right)^2\right)-\frac{1}{b^4}\left(\frac{db}{dx}\right)^4\right]}{\left[1+\frac{1}{b^2}\left(\frac{db}{dx}\right)^2x(1-x)\right]^2}\right.$$

$$T = \frac{a}{4b}\left\{1 + x^2(1-x)^2\frac{\left[\left(-\frac{[\;]}{a}+\frac{\frac{\partial^2 a}{\partial x^2}}{a}\right)\left(\frac{[\;]}{a}-2\frac{da}{a\,dx}\frac{db}{b\,dx}+\frac{4}{b^2}\left(\frac{db}{dx}\right)^2\right)-\frac{1}{b^4}\left(\frac{db}{dx}\right)^4\right]}{\left[1+\frac{1}{b^2}\left(\frac{db}{dx}\right)^2 x(1-x)\right]^2}\right\}$$

Is $\frac{d^2\frac{a}{b}}{dx^2}=0$ dan is $\frac{\partial^2 a}{\partial x^2} = \frac{2}{b}\frac{da}{dx}\frac{db}{dx} - \frac{2a}{b^2}\left(\frac{db}{dx}\right)^2$

of $-\frac{[\;]}{a}+\frac{\partial^2 a}{\partial x^2}\frac{1}{a} = -\frac{1}{a}\frac{\partial^2 a}{\partial x^2}+\frac{1}{a^2}\left(\frac{da}{dx}\right)^2 = -\frac{2}{ab}\frac{da}{dx}\frac{db}{dx}+\frac{2}{b^2}\left(\frac{db}{dx}\right)^2+\frac{1}{a^2}\left(\frac{da}{dx}\right)^2$

en $\frac{[\;]}{a}-2\frac{da}{a\,dx}\frac{db}{b\,dx}+\frac{4}{b^2}\left(\frac{db}{dx}\right)^2 = \frac{2}{ab}\frac{da}{dx}\frac{db}{dx}$ [doorgehaald] $-\frac{1}{a^2}\left(\frac{da}{dx}\right)^2$

Dus Teller $= -\left(\frac{1}{a^2}\left(\frac{da}{dx}\right)^2-\frac{2}{ab}\frac{da}{dx}\frac{db}{dx}+\frac{2}{b^2}\left(\frac{db}{dx}\right)^2\right)\left(\frac{1}{a^2}\left(\frac{da}{dx}\right)^2-\frac{2}{ab}\frac{da}{dx}\frac{db}{dx}\right)+\frac{1}{b^4}\left(\frac{db}{dx}\right)^4$

of $-\left\{\frac{1}{b^4}\left(\frac{db}{dx}\right)^4+\frac{2}{b^2}\left(\frac{db}{dx}\right)^2\left[\frac{1}{a^2}\left(\frac{da}{dx}\right)^2-\frac{2}{ab}\frac{da}{dx}\frac{db}{dx}\right]+\left[\frac{1}{a^2}\left(\frac{da}{dx}\right)^2-\frac{2}{ab}\frac{da}{dx}\frac{db}{dx}\right]^2\right\}$

$-\left\{\frac{1}{b^2}\left(\frac{db}{dx}\right)^2-\frac{2}{ab}\frac{da}{dx}\frac{db}{dx}+\frac{1}{a^2}\left(\frac{da}{dx}\right)^2\right\}^2 = -\left\{\frac{1}{b}\frac{db}{dx}-\frac{1}{a}\frac{da}{dx}\right\}^4$

$$T = \frac{a}{4b}\left\{1 - x^2(1-x)^2\frac{\left(\frac{1}{a}\frac{da}{dx}-\frac{1}{b}\frac{db}{dx}\right)^4}{\left\{1+\frac{1}{b^2}\left(\frac{db}{dx}\right)^2x(1-x)\right\}^2}\right\}$$

$$1-4\frac{T}{\frac{a}{b}} = \frac{\left(\frac{1}{a}\frac{da}{dx}-\frac{1}{b}\frac{db}{dx}\right)^4}{\left\{\frac{1}{x(1-x)}+\frac{\left(\frac{db}{dx}\right)^2}{b^2}\right\}^2} = \frac{\left(\frac{d\frac{a}{b}}{dx}\right)^4}{\frac{a^4}{b^8}\left\{\frac{b^2}{x(1-x)}+\left(\frac{db}{dx}\right)^2\right\}^2} = \frac{\frac{1}{\left(\frac{a}{b^2}\right)^4}\left(\frac{d\frac{a}{b}}{dx}\right)^4}{\left\{\frac{b^2}{x(1-x)}+\left(\frac{db}{dx}\right)^2\right\}^2}$$

$$\frac{\frac{4T}{\frac{a^2}{b^2}}\frac{d\frac{a}{b}}{dx}}{1-4\frac{T}{\frac{a}{b}}} = -4\frac{\frac{d\frac{a}{b^2}}{dx}}{\frac{a}{b^2}}-2\frac{\frac{d\frac{b^2}{x(1-x)}}{dx}}{\frac{b^2}{x(1-x)}+\left(\frac{db}{dx}\right)^2}$$

$$\frac{-\frac{4T}{\frac{a}{b}}\frac{d\frac{a}{b}}{dx}}{1-\frac{4T}{a/b}} = +4b\frac{d\frac{a}{b^2}}{dx}+2\frac{a}{b}\frac{\frac{d\frac{b^2}{x(1-x)}}{dx}}{\frac{b^2}{x(1-x)}+\left(\frac{db}{dx}\right)^2}$$

of $\frac{\left(1-\frac{4T}{\frac{a}{b}}-1\right)}{1-\frac{4T}{\frac{a}{b}}}\frac{d\frac{a}{b}}{dx} = 4b\frac{d\frac{a}{b^2}}{dx}+2\frac{a}{b}\frac{\frac{d\frac{b^2}{x(1-x)}}{dx}}{\frac{b^2}{x(1-x)}+\left(\frac{db}{dx}\right)^2}$

of

$$\frac{+\frac{d\frac{a}{b}}{dx}}{1-\frac{4T}{\frac{a}{b}}} = +\frac{d\frac{a}{b}}{dx}-4b\frac{d\frac{a}{b^2}}{dx}-2\frac{a}{b}\frac{\frac{d\frac{b^2}{x(1-x)}}{dx}}{\frac{b^2}{x(1-x)}+\left(\frac{db}{dx}\right)^2}$$

$$1-\frac{4T}{\frac{a}{b}} = \frac{\frac{d\frac{a}{b}}{dx}}{\frac{d\frac{a}{b}}{dx}-4b\frac{d\frac{a}{b^2}}{dx}-2\frac{a}{b}\frac{\frac{d\frac{b^2}{x(1-x)}}{dx}}{\frac{b^2}{x(1-x)^2}+\left(\frac{db}{dx}\right)^2}}$$

of $1 - \frac{4T}{\frac{a}{b}} = \left(\frac{1}{\frac{a}{b^2}}\right)^4 \frac{\left(\frac{d\frac{a}{b}}{dx}\right)^4}{\left\{\frac{b^2}{x(1-x)} + \left(\frac{db}{dx}\right)^2\right\}^2} = \frac{\frac{d\frac{a}{b}}{dx}}{\frac{d\frac{a}{b}}{dx} - 4b\frac{d\frac{a}{b^2}}{dx} - 2\frac{a}{b}\frac{\frac{d\frac{b^2}{x(1-x)}}{dx}}{\frac{b^2}{x(1-x)} + \left(\frac{db}{dx}\right)^2}}$

of $\frac{1}{\left(\frac{a^2}{b^4}\right)^4}\left(\frac{d\frac{a}{b}}{dx}\right)^3\left\{\frac{d\frac{a}{b}}{dx} - 4b\frac{d\frac{a}{b^2}}{dx} - 2\frac{a}{b}\frac{\frac{d\frac{b^2}{x(1-x)}}{dx}}{\frac{b^2}{x(1-x)} + \left(\frac{db}{dx}\right)^2}\right\} = \frac{\left\{\frac{b^2}{x(1-x)} + \left(\frac{db}{dx}\right)^2\right\}^2}{b^4}$

$\frac{\left(\frac{d\frac{a}{b}}{dx}\right)^3}{\left(\frac{a}{b}\right)^3}\left\{\frac{-3\frac{d\frac{a}{b}}{dx}}{\frac{a}{b}} + 4\frac{db}{b\,dx} - 2\frac{\frac{d\left[\frac{b^2}{x(1-x)} + \left(\frac{db}{dx}\right)^2\right]}{dx}}{\frac{b^2}{x(1-x)} + \left(\frac{db}{dx}\right)^2}\right\} = \left\{\frac{1}{x(1-x)} + \frac{1}{b^2}\left(\frac{db}{dx}\right)^2\right\}^2$

$\frac{\left(\frac{d\left(\frac{a}{b}\right)}{dx}\right)^3}{\left(\frac{a}{b}\right)^3}\left\{-\frac{3\frac{d\frac{a}{b}}{dx}}{\frac{a}{b}} - 2\frac{\frac{d\left[\frac{1}{x(1-x)} + \frac{1}{b^2}\left(\frac{db}{dx}\right)^2\right]}{dx}}{\frac{1}{x(1-x)} + \frac{1}{b^2}\left(\frac{db}{dx}\right)^2}\right\} = \left\{\frac{1}{x(1-x)} + \frac{1}{b^2}\left(\frac{db}{dx}\right)^2\right\}^2$

$-\frac{3B^4}{(A+Bx)^4} - 2\frac{B^3}{(A+Bx)^3}\frac{du}{u\,dx} = u^2$

$u = (A+Bx)^n K$

$-\frac{3B^4}{(A+Bx)^4} - \frac{2B^3 \cdot nB}{(A+Bx)^3(A+Bx)} = (A+Bx)^{2n} K^2$

$2n = -4$

$K^2 = -B^4(3-4)$

$K^2 = B^4$

~~$\frac{u}{(A+Bx)^2} = \pm\left(\frac{d\frac{a}{b}}{dx}\right)^2$~~

$\frac{a_x}{b_x} = \frac{a_1}{b_1}(1-x) + \frac{a_2}{b_2}x + b_1 b_2\left(\frac{2a_{12}}{b_1 b_2} - \frac{a_1}{b_1^2} - \frac{a_2}{b_2^2}\right) \times \frac{x(1-x)}{b_1(1-x) + b_2 x}$

$\frac{d\frac{a_x}{b_x}}{dx} = \frac{a_2}{b_2} - \frac{a_1}{b_1} + b_1 b_2\left(\frac{2a_{12}}{b_1 b_2} - \frac{a_1}{b_1^2} - \frac{a_2}{b_2^2}\right)\left\{\frac{1-2x}{b_x} - \frac{x(1-x)}{b_x^2}\frac{db_x}{dx}\right\}$

$\frac{d^2\frac{a}{b}}{dx^2} = -2\frac{b_1^2 b_2^2}{b_x^3}\left(\frac{2a_{12}}{b_1 b_2} - \frac{a_1}{b_1^2} - \frac{a_2}{b_2^2}\right)$

If $\frac{a}{b} = A + Bx$

ln $\frac{1}{x(1-x)} + \frac{1}{b^2}\left(\frac{db}{dx}\right)^2 = u$

~~of $u = \pm\frac{d\frac{a}{b}}{dx}\left(\frac{a}{b}\right)^2$~~

~~$u = \pm\frac{1}{b}$~~

$3\int u\,dx = C - \left(\frac{a}{b}\right)^3$

of $u = \left(\frac{a}{b}\right)^{-2}\left(\frac{d\frac{a}{b}}{dx}\right)^2$

$u = \left(\frac{d\log\frac{a}{b}}{dx}\right)^2$

$$1 - \frac{4T}{\frac{a}{b}} = \frac{\left(\frac{d\frac{a}{b}}{dx}\right)^4}{\left[\left(\frac{a}{b}\right)^2 \left\{ \frac{1}{x(1-x)} + \left(\frac{db}{dx}\right)^2 \frac{1}{b^2} \right\}\right]^2} = W \left(\frac{d\frac{a}{b}}{dx}\right)^4$$

$$\frac{4T}{\frac{a^2}{b^2}} \frac{d\frac{a}{b}}{dx} = \left(\frac{d\frac{a}{b}}{dx}\right)^4 \frac{dW}{dx} \quad \text{or} \quad 1 = \frac{a}{b}\left(\frac{d\frac{a}{b}}{dx}\right)^3 \frac{dW}{dx} + W \left(\frac{d\frac{a}{b}}{dx}\right)^4$$

$$\frac{4T}{\frac{a}{b}} = \frac{a}{b}\left(\frac{d\frac{a}{b}}{dx}\right)^3 \frac{dW}{dx} \qquad \frac{1}{\left(\frac{d\frac{a}{b}}{dx}\right)^3} = \frac{d\left(\frac{a}{b}W\right)}{dx}$$

$$\frac{a}{b} = \frac{a_1}{b_1} + x\left(\frac{a_2}{b_2} - \frac{a_1}{b_1}\right) \qquad \frac{1}{\left(\frac{a_2}{b_2} - \frac{a_1}{b_1}\right)^3} =$$

(struck through)

$$1 - \frac{4T}{\frac{a}{b}} = \frac{a}{b}\left(\frac{d\frac{a}{b}}{dx}\right)^3 \frac{dW}{dx} - W\left(\frac{d\frac{a}{b}}{dx}\right)^4 = \left(\frac{d\frac{a}{b}}{dx}\right)^3 \left\{ \frac{a}{b}\frac{dW}{dx} - W \frac{d\frac{a}{b}}{dx} \right.$$

$$W = \frac{1}{\frac{a^4}{b^4}\left\{ \frac{1}{x(1-x)} + \frac{1}{b^2}\left(\frac{db}{dx}\right)^2 \right\}^2}$$

$$\left(\frac{a}{b}\right) W = \frac{1}{\frac{a^3}{b^3}\left\{ \frac{1}{x(1-x)} + \frac{1}{b^2}\left(\frac{db}{dx}\right)^2 \right\}^2} =$$

(struck through)

$$\frac{1}{\left(\frac{d\frac{a}{b}}{dx}\right)^3} = -3 \frac{\frac{d\frac{a}{b}}{dx}}{\left(\frac{a^4}{b^4}\right)} \frac{1}{u} - \frac{\frac{du}{dx}}{\frac{a^3}{b^3} u^2}$$

$$\frac{a}{b} W = \frac{1}{\left(\frac{a_1}{b_1}(1-x) + \frac{a_2}{b_2} x\right)^3 \left\{ \frac{1}{x(1-x)} + \frac{(b_2 - b_1)^2}{(b_1(1-x) + b_2 x)^2} \right\}^2}$$

$$\left(\frac{MKT}{x(1-x)} - \frac{\partial^2 a}{\partial x^2}\frac{1}{v}\right)\left(\frac{MKT}{(v-b)^2} - \frac{2a}{v^3}\right) - \frac{2a}{v^3}\frac{MKT}{(v-b)^2}\left(\frac{\partial b}{\partial x}\right)^2 + 2\frac{da}{dx}\frac{1}{v^2}$$

Lq $v = nb$

$$\left(\frac{MKT}{x(1-x)} - \frac{\partial^2 a}{\partial x^2}\frac{1}{nb}\right)\left(\frac{MKT}{(n-1)^2} - \frac{2a}{n^3 b}\right)\frac{1}{b^2} - \frac{2a}{n^3}\frac{MKT}{(n-1)^2}\frac{\left(\frac{db}{dx}\right)^2}{b^3}$$

$$\left(\frac{MKT}{x(1-x)} - \frac{\partial^2 a}{\partial x^2}\frac{1}{nb}\right)\left\{1 - \frac{2a(n-1)^2}{MKTb\,n^3}\right\} - \frac{2a}{n^3}\frac{\left(\frac{db}{dx}\right)^2}{b^3}$$

$$\frac{MKT}{x(1-x)} - \frac{\partial^2 a}{\partial x^2}\frac{1}{nb} + \frac{2a}{n^2b^2}\frac{da}{dx}\frac{db}{dx} - \frac{2a}{n^3}\left(\frac{db}{dx}\right)^2\frac{1}{b^3} - \frac{(n-1)^2}{n^3b^2}\Big\{$$

$$\frac{MKT}{x(1-x)} - \frac{\partial^2 a}{\partial x^2}\frac{1}{v} + \frac{2a}{v^2}\frac{da}{dx}\frac{dv}{dx} - \frac{2a}{v^3}\left(\frac{dv}{dx}\right)^2\frac{1}{v^3}$$

Lq $2a\frac{\partial^2 a}{\partial x^2} - \left(\frac{da}{dx}\right)^2 = 0$

$$MKT + \frac{2a}{b}\frac{(n-1)^2}{n^3} = \frac{x(1-x)}{2a}\left\{\frac{1}{b}\frac{\partial^2 a}{\partial x^2} - \frac{4a}{nb}\frac{da}{dx}\frac{db}{dx}\right.$$

$$2\frac{d\frac{a}{b}}{dx}\frac{(n-1)^2}{n^3} = \frac{x(1-x)}{n}\left\{-\frac{1}{b^2}\frac{\partial^2 a}{\partial x^2}\frac{db}{dx} - \frac{2}{nb^2}\frac{db}{dx}\frac{\partial^2 a}{\partial x^2}\right.$$

$$+\frac{1-2x}{n}\left\{\frac{1}{b}\frac{\partial^2 a}{\partial x^2} - \frac{2}{nb^2}\frac{da}{dx}\frac{db}{dx}\right.$$

$$\frac{MKT}{x(1-x)} - \frac{2a(n-1)^2}{b^3n^3} = \frac{1}{n}\left\{\frac{d^2\frac{a}{b}}{dx^2} + 2\frac{n-1}{n}\frac{1}{b}\frac{db}{dx}\frac{d\frac{a}{b}}{dx}\right.$$

$$MKT + \frac{2a}{b}\frac{(n-1)^2}{n^3} = \frac{x(1-x)a}{2abn}\left\{\frac{1}{a}\frac{da}{dx} - \frac{2}{nb}\left(\frac{db}{dx}\right)\right\}^2$$

$$MKT + \frac{2(n-1)^2}{n^3}\frac{a}{b} = \frac{x(1-x)}{2n}\frac{a}{b}\left\{\frac{1}{a}\left(\frac{da}{dx}\right) - \frac{2}{nb}\left(\frac{db}{dx}\right)\right\}^2$$

$$\frac{MKT}{(v-b)^2}\frac{db}{dx} - \left(\frac{da}{dx}\right)^2\frac{1}{v^4} = 0$$

$$+ 2\frac{da}{dx}\frac{1}{n^2b^2}\frac{MKT}{(n-1)^2}\frac{db}{dx} - \left(\frac{da}{dx}\right)^2\frac{1}{n^4b^2} = 0$$

$$+ 2\frac{da}{dx}\frac{1}{n^2b^2}\frac{db}{dx} - \left(\frac{da}{dx}\right)^2\frac{(n-1)^2}{n^4b^2\,MKT} = 0$$

$$-\frac{2a(n-1)^2}{bn^3\,x(1-x)}$$

$$-\frac{2a(n-1)^2}{b\,n^3\,x(1-x)} \ast \left(\frac{2a\frac{\partial^2 a}{\partial x^2} - \left(\frac{da}{dx}\right)^2}{MKT}\right)\left\{\frac{(n-1)^2}{n^4b^2}\right.$$

$$\left(\frac{da}{dx}\right)^2$$

$$+\frac{4a^2}{n^2b^3}\left(\frac{db}{dx}\right)^2\Big\} = \frac{x(1-x)}{nb\,2a}\left\{\frac{\partial^2 a}{\partial x^2} - \frac{4a}{nb}\frac{da}{dx}\frac{db}{dx} - \frac{4a^2}{n^2}\left(\frac{db}{dx}\right)^2\frac{1}{b^2}\right\}$$

$$+\frac{2}{nb^3}\frac{da}{dx}\left(\frac{db}{dx}\right)^2 + \frac{2}{n^2b^3}\frac{da}{dx}\left(\frac{db}{dx}\right)^2 - \frac{2}{n^2b^4}\left(\frac{db}{dx}\right)^3\Big\}$$

$+\frac{2a}{n^2b^3}\left(\frac{db}{dx}\right)^2\Big\}$ Lq $MKT + \frac{2a}{b}\frac{(n-1)^2}{n^3} = \frac{x(1-x)}{2anb}\left(\left(\frac{da}{dx}\right) - \frac{2a}{n}\frac{db}{dx}\right)^2$

$$-\frac{2(n-1)}{n^2}\frac{a}{b^3}\left(\frac{db}{dx}\right)^2\Big\}$$

$$\left(\frac{T}{x(1-x)} - \frac{\partial^2 a}{\partial x^2}\frac{1}{v}\right)\left(\frac{T}{(v-b)^2} - \frac{2a}{v^3}\right) - \frac{2a}{v^3}\frac{T}{(v-b)^2}\left(\frac{db}{dx}\right)^2 + \frac{2}{v^2}\frac{T}{(v-b)^2}\frac{da}{dx}\frac{db}{dx}$$

$$\frac{T^2}{x(1-x)} - \frac{2aT}{x(1-x)}\frac{(v-b)^2}{v^3} - T\frac{\partial^2 a}{\partial x^2}\frac{1}{v} - \frac{2a}{v^3}T\left(\frac{db}{dx}\right)^2 + \frac{2T}{v^2}\frac{da}{dx}\frac{db}{dx}$$

$$2a\frac{d^2a}{dx^2} = \left(\frac{da}{dx}\right)^2$$

$$Tv^3 - 2a(v-b)^2 - \frac{x(1-x)}{2a}\left\{\frac{1}{2a}\left(\frac{da}{dx}\right)^2 v^2 - 4va\frac{da}{dx}\frac{db}{dx} + 4a^2\left(\frac{db}{dx}\right)^2\right\}$$

$$(v-b)\frac{da}{dx} + \left(b\frac{da}{dx} - 2a\frac{db}{dx}\right)$$

$$Tv^3 = 2a(v-b)^2 + \frac{x(1-x)}{2a}\left\{v\frac{da}{dx} - 2a\frac{db}{dx}\right\}^2$$

$$3Tv^2 = 4a(v-b) + 2\frac{x(1-x)}{2a}\frac{da}{dx}\left(v\frac{da}{dx} - 2a\frac{db}{dx}\right)$$

$$Tv^3 = (v-b)^2\left\{2a + \frac{x(1-x)}{2a}\left(\frac{da}{dx}\right)^2\right\} + (v-b)\frac{x(1-x)}{a}\frac{da}{dx}\left(b\frac{da}{dx} - 2a\right.$$

$$3Tv^2 = 2(v-b)\left\{2a + \frac{x(1-x)}{2a}\left(\frac{da}{dx}\right)^2\right\} + \frac{x(1-x)}{a}\frac{da}{dx}\left(b\frac{da}{dx} - 2a\right.$$

$$0 = (v-3b)(v-b)\left\{2a + \frac{x(1-x)}{2a}\left(\frac{da}{dx}\right)^2\right\} + (2v-3b)\frac{x(1-x)}{a}\frac{da}{dx}\left(b\frac{da}{dx} - \right.$$

is $\frac{da}{a\,dx} = 2\frac{db}{b\,dx}$ of $\frac{d\frac{a}{b^2}}{dx} = 0$ dan $v = 3b$

$$T = 2a\frac{4b^2}{27b^3} + \frac{x(1-x)}{2a}\left\{3b\frac{da}{dx} - 2a\frac{db}{dx}\right\}^2$$

$$T = \frac{8a}{27b} + \frac{x(1-x)}{2a}\,4a\frac{db}{dx}\,\frac{16a^2\left(\frac{db}{dx}\right)^2}{27b^3}$$

$$T = \frac{8}{27}\frac{a}{b}\left\{1 + \frac{x(1-x)}{b^2}\left(\frac{db}{dx}\right)^2\right\}$$

$$v = 2b\,\frac{1+\frac{x(1-x)}{2ab}\frac{da}{dx}\frac{db}{dx}}{1+\frac{x(1-x)}{4a^2}\left(\frac{da}{dx}\right)^2}\left\{1 + \frac{1}{2}\sqrt{\left[1 - \frac{3x(1-x)\left[\frac{da}{2a\,dx} - \frac{db}{b\,dx}\right]^2}{\left(1+\frac{x(1-x)}{2ab}\frac{da}{dx}\frac{db}{dx}\right)^2}\right]}\right\}$$

$$- \left(\frac{da}{dx}\right)^2\frac{(v-b)^2}{v^4} = 0$$

$$+ \frac{(v-b)^2}{v^4}\left[2a\frac{d^2a}{dx^2} - \left(\frac{da}{dx}\right)^2\right] = 0$$

$$= 0$$

$$\left.\frac{db}{dx}\right) + \frac{x(1-x)}{2a}\left(b\frac{da}{dx} - 2a\frac{db}{dx}\right)^2$$

$$\left.\frac{db}{dx}\right)$$

$$\left.2a\frac{db}{dx}\right) + \frac{3x(1-x)}{2a}\left(b\frac{da}{dx} - 2a\frac{db}{dx}\right)^2$$

$$v = \frac{2b\left\{1 + \frac{x(1-x)}{2ab}\frac{da}{dx}\frac{db}{dx}\right\} \pm b\sqrt{\left\{1 + \frac{x(1-x)}{4a^2}\left(\frac{da}{dx}\right)^2 - x(1-x)\left(\frac{da}{a\,dx} - \frac{db}{b\,dx}\right)\left(\frac{da}{a\,dx} - \frac{3db}{b\,dx}\right) + \frac{x^2(1-x)^2}{4a^2b^2}\left(\frac{da}{dx}\right)^2\left(\frac{db}{dx}\right)^2\right\}}}{1 + \frac{x(1-x)}{4a^2}\left(\frac{da}{dx}\right)^2}$$

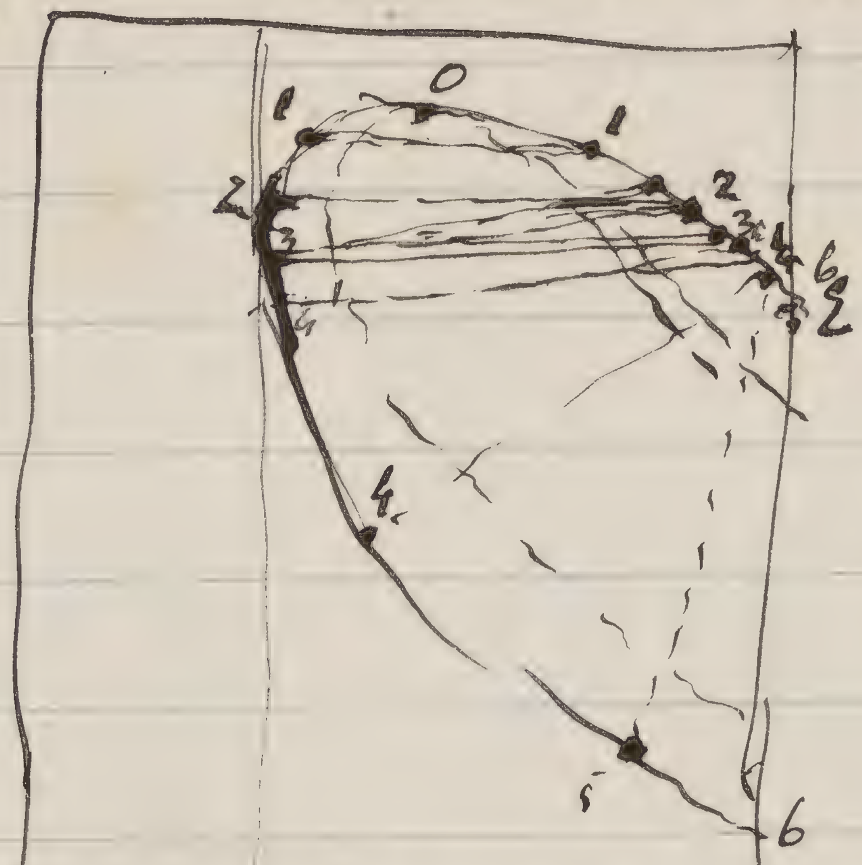
$$v^2\left\{1 + \frac{x(1-x)}{4a^2}\left(\frac{da}{dx}\right)^2\right\} - 4bv\left\{1 + \frac{x(1-x)}{2ab}\frac{da}{dx}\frac{db}{dx}\right\} + 3b^2\left(1 + \frac{x(1-x)}{b^2}\left(\frac{db}{dx}\right)^2\right) = 0$$

is $\frac{1}{2a}\frac{da}{dx} = \frac{db}{b\,dx}$ dan $v^2 - 4bv + 3b^2 = 0$

of $\frac{d\frac{a}{b^2}}{dx} = 0$ [illegible]

$$\text{of } v = \frac{2b\left\{1 + \frac{x(1-x)}{2ab}\frac{da}{dx}\frac{db}{dx}\right\} \pm b\sqrt{\left\{\left[1 + \frac{x(1-x)}{2ab}\frac{da}{dx}\frac{db}{dx}\right]^2 - 3x(1-x)\left[\frac{da}{2a\,dx} - \frac{db}{b\,dx}\right]\right\}}}{1 + \frac{x(1-x)}{4a^2}\left(\frac{da}{dx}\right)^2}$$

ClH en CO_2
CO_2 0   1   2 × ClH

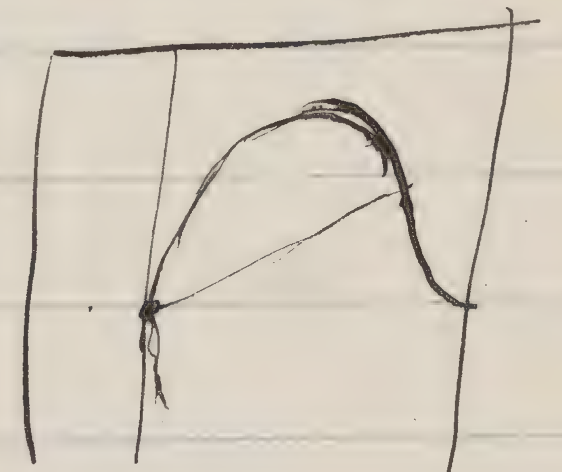


$\frac{dp}{dx_2}\left\{\frac{v_2-v_1}{x_2-x_1}-\left(\frac{dv_2}{dx_2}\right)_p\right\}=+$

$\frac{dp}{dx_2}$ is negatief en dus

$\frac{v_2-v_1}{x_2-x_1}-\left(\frac{dv_2}{dx_2}\right)_p$ negatief

Naar v_2-v_1 reeds negatief tot in ~~bij~~ 3 maar daar is $v_2-v_1=0$ en ~~bij~~ 1 en 0 2 zelfs positief. Over het geheele beloop zal $\frac{dv_2}{dx_2}>0$ zijn.



$\frac{dp}{dx_1}$ is ~~positief~~ negatief ~~van~~ 6 en 4 is

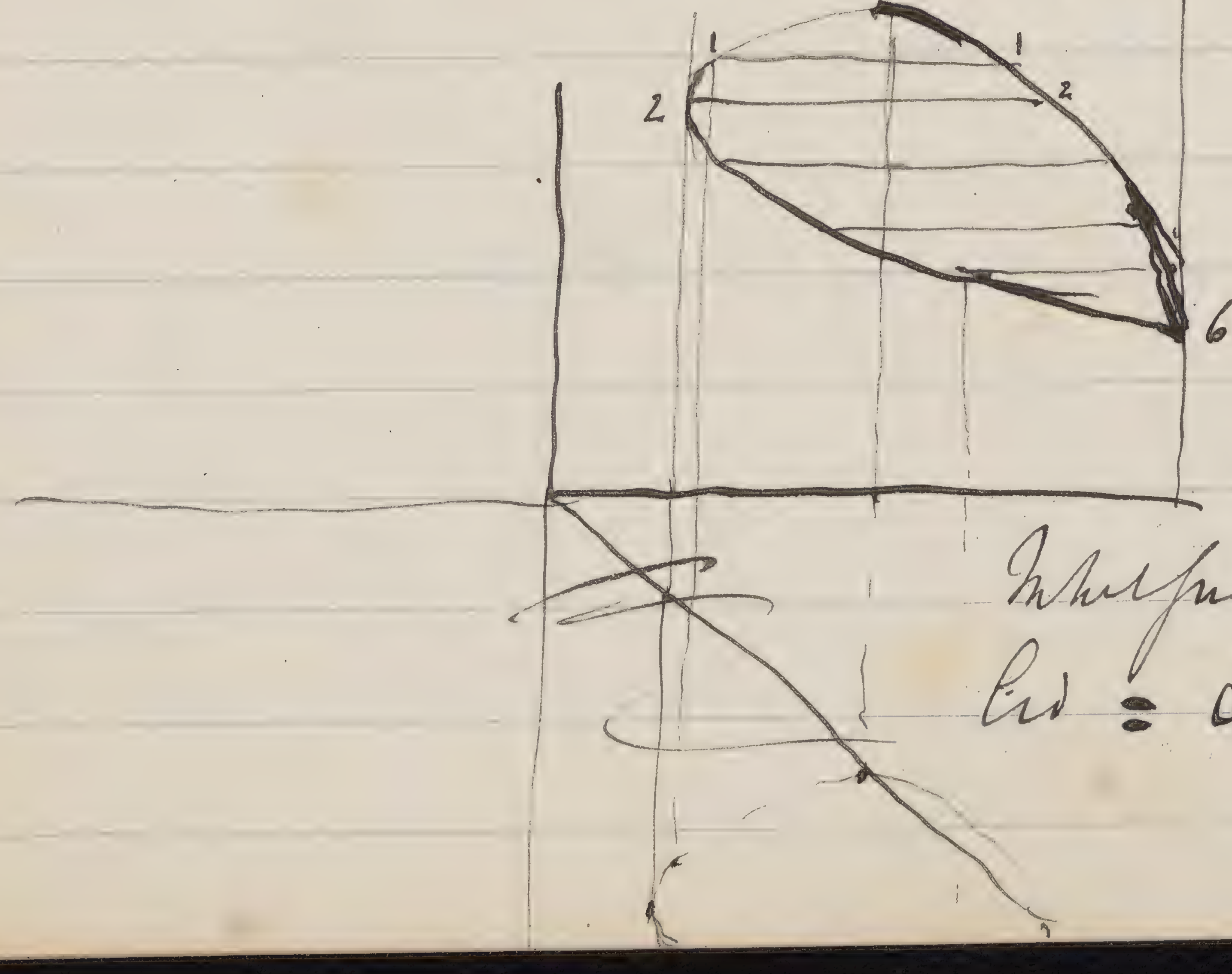
en dus $\frac{v_2-v_1}{x_2-x_1}-\left(\frac{dv_1}{dx_1}\right)_p$ negatief.

In 3 is $v_2=v_1$ en dus ook $\left(\frac{dv_1}{dx_1}\right)_p$ positief

~~In 2 = $\left(\frac{dv_1}{dx_1}\right)=\frac{v_2-v_1}{x_2-x_1}$ en dus $\left(\frac{dp}{dx_2}\right)=-\infty$~~

~~In 1 $\left(\frac{dv_1}{dx_1}\right) > \frac{v_2-v_1}{x_2-x_1}$~~ voorbij 2 in $\left(\frac{dp}{dx_2}\right)=+\infty$

En in 1 positief of $\frac{v_2-v_1}{x_2-x_1}<\left(\frac{dv_1}{dx_1}\right)_p$

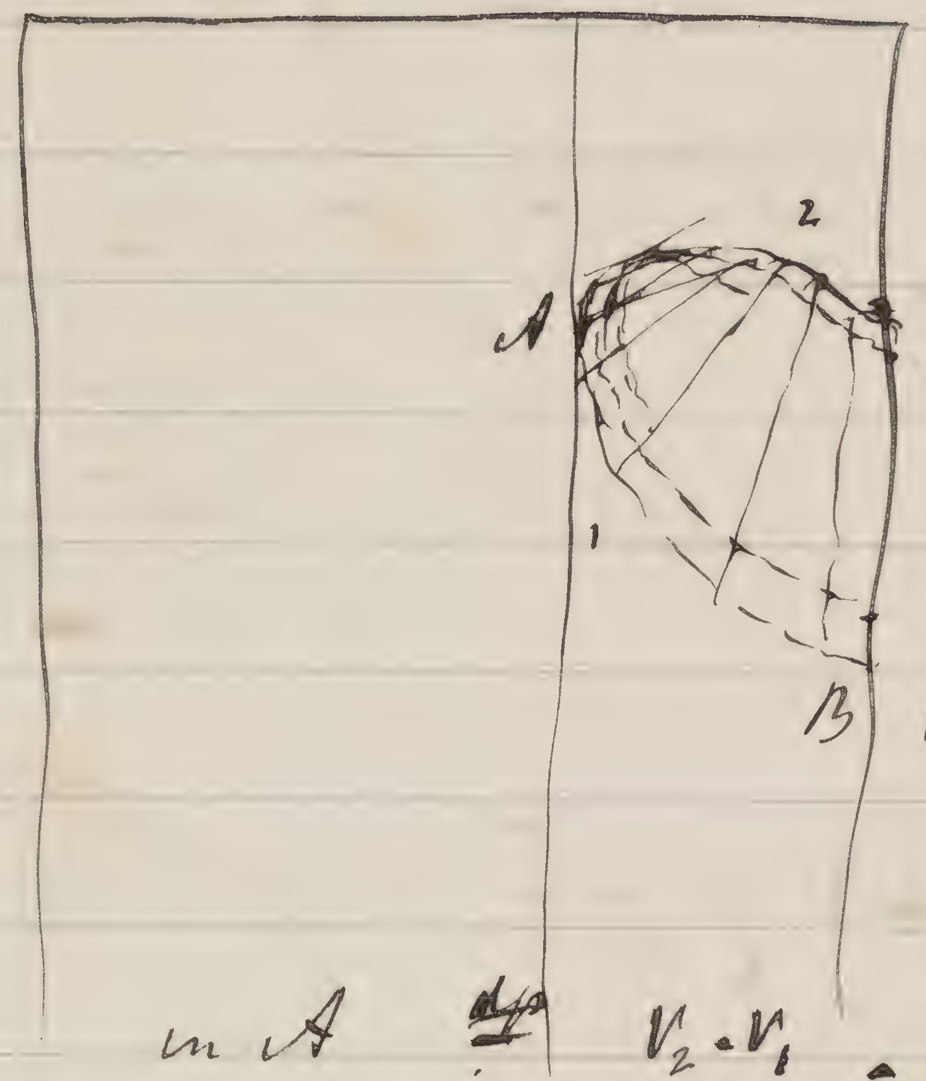


Het punt 2 kritisch punt van het mengsel 7 CO_2 3 ClH (T = 31°6 Druk 90 Atm

In het punt nul is het tweede lid = 0 en dus dp = 0

Lucht CO_2 9 CO_2 1 Lucht

25°   Druk 77.5



$\frac{dp}{dx_2}$ steeds negatief

of $\frac{v_2-v_1}{x_2-x_1}-\left(\frac{dv_2}{dx_2}\right)_p$ negatief

Reeds v_2-v_1 is negatief

en $\left(\frac{dv_2}{dx_2}\right)=\frac{\frac{MRT}{v-b}\frac{db}{dx}-\frac{da}{dx}\frac{1}{v^2}}{\frac{MRT}{v-b}-\frac{a}{v^2}}$ is positief

in A $\frac{dp}{dx_1}$ $\frac{v_2-v_1}{x_2-x_1}=\left(\frac{dv_1}{dx_1}\right)_p$

tusschen A en B $\frac{v_2-v_1}{x_2-x_1}$ negatief en $\frac{v_2-v_1}{x_2-x_1}-\left(\frac{dv_1}{dx_1}\right)_p$ negatief

Dus de listing dat v_2 steeds < v_1 is bevestigende voorstelling dat $\frac{dv_1}{dx_1}$ negatief kan zijn – ~~voordat voorbij A $\frac{v_2-v_1}{x_2-x_1}$~~ – Anders voorbij A $\frac{dp}{dx_1}$ positief is en dus $\frac{v_2-v_1}{x_2-x_1}-\frac{dv_1}{dx_1}$ positief moet zijn. Is nu v_2 steeds < v_1, dan is dus niet mogelijk. Maar $\left(\frac{dv_1}{dx_1}\right)_p$ negatief is niet waarschijnlijk onder een druk van 77½ Atm. Ik bovendien dat de bij volumes op den medeopgaanden tak der drukking. Dus bij A is v_1 reeds grooter dan v_2 evenals bij ~~de~~ afgenomen vorige bladzijde

Loop der spinodale lijn

$$\left(\frac{MRT}{x(1-x)}-\frac{1}{V}\frac{\partial^2 a}{\partial x^2}\right)\left(\frac{MRT}{(V-b)^2}-\frac{2a}{V^3}\right)-\frac{MRT}{(V-b)^2}\left\{\frac{2a}{V^3}\left(\frac{db}{dx}\right)^2-2\frac{1}{V^2}\frac{da}{dx}\frac{db}{dx}\right\}-\left(\frac{da}{dx}\right)^2\frac{1}{V^4}=0$$

of

$$\left\{\frac{MRT}{x(1-x)}-\frac{1}{V}\left[\frac{\partial^2 a}{dx^2}-\frac{1}{2a}\left(\frac{da}{dx}\right)^2\right]\right\}\left\{\frac{MRT}{(V-b)^2}-\frac{2a}{V^3}\right\}-\frac{MRT}{(V-b)^2}\left\{\frac{1}{2aV}\left(\frac{da}{dx}\right)^2-\frac{2}{V^2}\frac{da}{dx}\frac{db}{dx}+\frac{2a}{V^3}\left(\frac{db}{dx}\right)^2\right\}=0$$

$$\left\{TV^3-2a(V-b)^2\right\}=\frac{\frac{x(1-x)}{2a}\left(V\frac{da}{dx}-2a\frac{db}{dx}\right)^2}{1-\frac{x(1-x)}{2aTV}\left[2a\frac{\partial^2 a}{\partial x^2}-\left(\frac{da}{dx}\right)^2\right]}$$

(T schrijvende voor MRT)

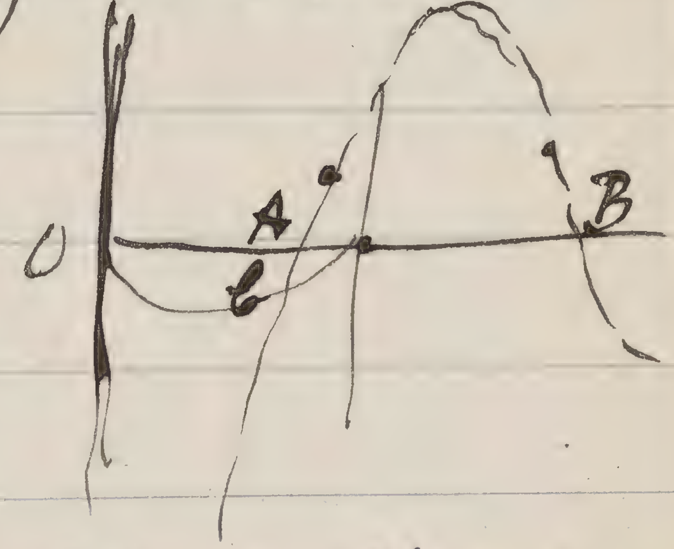
Als 4de machtsvergelijking in V

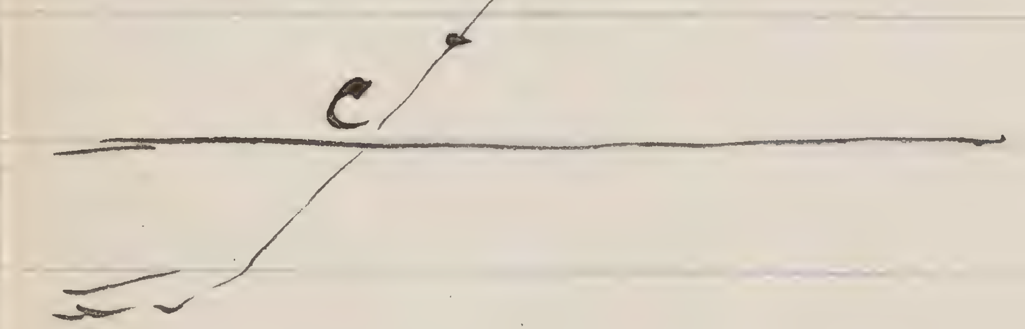
$$\frac{T^2V^4}{x(1-x)}-V^3\left\{\frac{2aT}{x(1-x)}+T\frac{\partial^2 a}{\partial x^2}\right\}+V^2\left\{\frac{4aTb}{x(1-x)}+2bT\frac{da}{dx}\frac{db}{dx}+2a\frac{\partial^2 a}{\partial x^2}-\left(\frac{da}{dx}\right)^2\right\}-V\left\{\frac{2aTb^2}{x(1-x)}+2aT\left(\frac{db}{dx}\right)^2+2b\left[2a\frac{\partial^2 a}{\partial x^2}-\left(\frac{da}{dx}\right)^2\right]\right\}+b^2\left\{2a\frac{\partial^2 a}{\partial x^2}-\left(\frac{da}{dx}\right)^2\right\}=0$$

Loop der lijn $TV^3-2a(V-b)^2=Y$

waarde in O $\quad -2ab^2$ } dus wortel

waarde in b $\quad Tb^3$ } tusschen O en b





tusschen B en C is Y negatief. Bij $T=\frac{\partial}{\partial x}\frac{a}{b}$ vallen B en C samen en eerst bij $9/8\,\frac{\partial}{\partial x}\,a/b$ is maximum en minimum aanwezig(?) ... kunnen verdwijnen.

1ste geval $\quad 2a\frac{\partial^2 a}{\partial x^2}-\left(\frac{da}{dx}\right)^2<0\quad$ of $4\left(a_1a_2-a_{12}^2\right)<0$

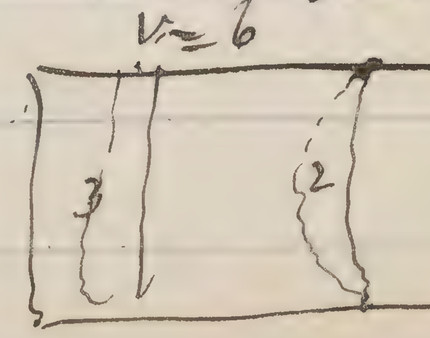
De Noemer $1-\frac{x(1-x)}{2aVT}\left[2a\frac{\partial^2 a}{\partial x^2}-\left(\frac{da}{dx}\right)^2\right]$ is dan steeds positief, en dus vallen de punten der spinodale lijn bij gekozen x voor positieve V op plaatsen waar Y positief is. Voor $x=0$ en $x=1$ zijn het de punten ABC zelven.

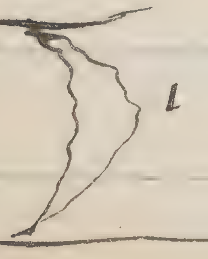
De punten B' en C' vallen dus verder uiteen of het labiele gebied is grooter dan bij een enkele stof.

Zoolang $\frac{T}{x(1-x)}>\frac{d^2 a}{dx^2}$ zal het punt B' verder dan b van O verwijderd zijn. Daar $\frac{d^2a}{dx^2}=2\frac{b_1^2b_2^2}{b^3}\left\{\left(\frac{\sqrt{a_1}}{b_1}-\frac{\sqrt{a_2}}{b_2}\right)^2-2\frac{a_{12}-\sqrt{a_1a_2}}{b_1b_2}\right\}$

en dus in het eerste geval negatief $\left(\text{als } a_{12}^2 > (a_{12}-\sqrt{a_1a_2}) \ldots\right)$. Bij elke waarde van T blijft dus de tak B' op het oppervlak kleiner dan b.

De derde tak heeft positieve waarde van V maar ... Bij derde 4de machtsvergelijking de laatste term negatief heeft is er steeds een wortel ... negatieve wortel. De viertakken, waarvan er twee beteekenis hebben, blijven dus gescheiden.





Is $\left(\frac{\sqrt{a_1}}{b_1}-\frac{\sqrt{a_2}}{b_2}\right)^2>2\frac{a_{12}-\sqrt{a_1a_2}}{b_1b_2}$ dan kan bij zeer lage T de tweede tak de lijn $V=b$ tweemaal snijden

$$TV^3 - 2a(V-b)^2 = \frac{\frac{x(1-x)}{2a}\left(V\frac{da}{dx} - 2a\frac{db}{dx}\right)^2}{1 - \frac{x(1-x)}{2aVT}\left[2a\frac{\partial^2 a}{\partial x^2} - \left(\frac{da}{dx}\right)^2\right]}$$

2de geval $\quad 2a\frac{\partial^2 a}{\partial x^2} - \left(\frac{da}{dx}\right)^2 = 4(a_1a_2 - a_{12}^2) > 0$

maar zoodat de noemer niet negatief kan worden voor waarden van $V > b$ of $bT > \frac{x(1-x)\,2(a_1a_2 - a_{12}^2)}{a}$

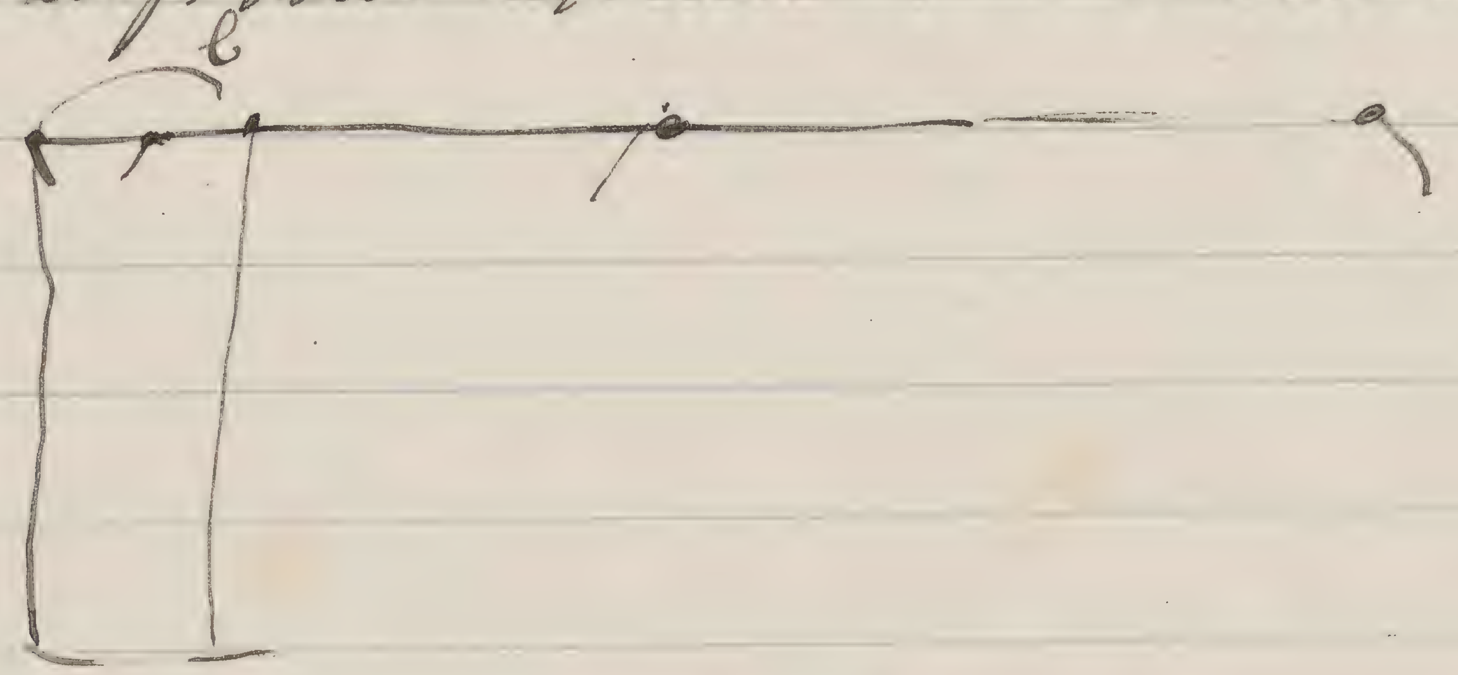
De maximumwaarde van $\frac{x(1-x)}{a}$ voor zekere waarde van x te vinden uit

$$(1-2x)\left[a_1 + 2(a_{12}-a_1)x + (a_2+a_1-2a_{12})x^2\right] - 2x(1-x)\left[(a_{12}-a_1)+\Delta x\right]$$

of $a_1(1-x)^2 = a_2x^2 \qquad \frac{x}{1-x} = \sqrt{\frac{a_1}{a_2}}$

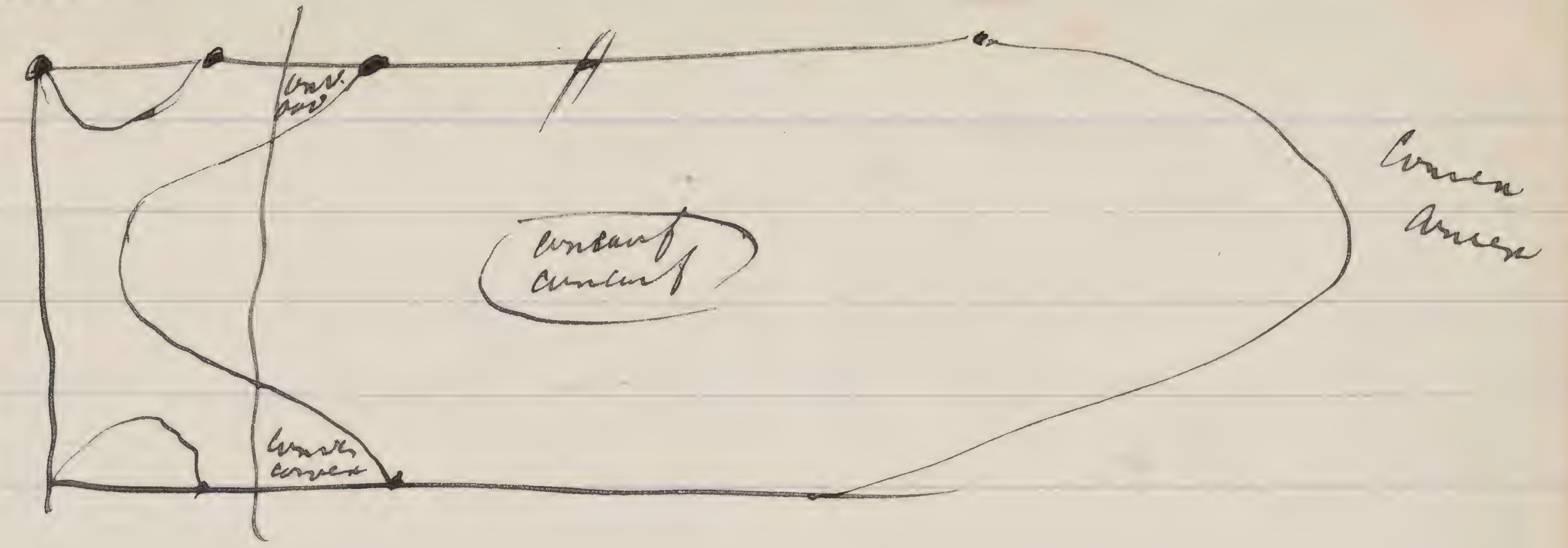
of $bT > \sqrt{a_1a_2} - a_{12}$

Nu zullen de 4 takken zóó moeten liggen dat het eerste + lid positief is. Zelfs geen negatieve. V is de noemer en b positief

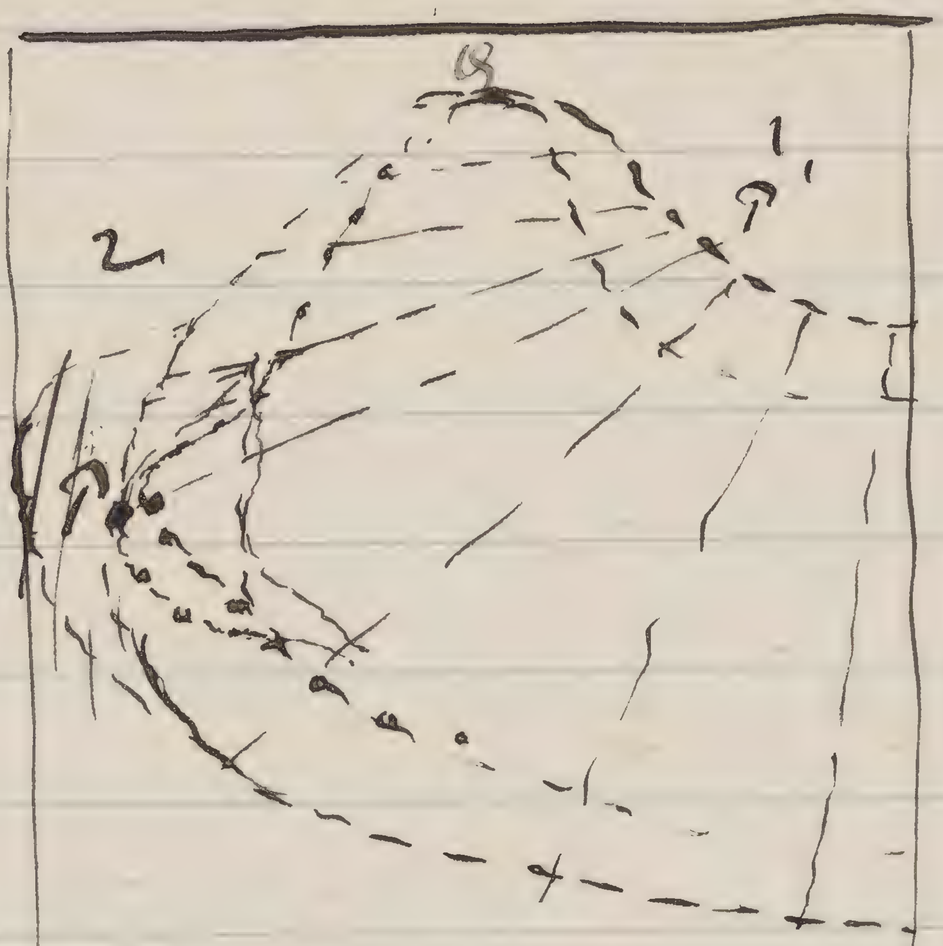


+ NB. Alleen kan de noemer negatief worden voor $V < b$. Ende de wortel der andere zijden $= 0$ is, behoort tot die waarde van V waarvoor het eerste lid negatief is

By lagere T $\quad V = b$



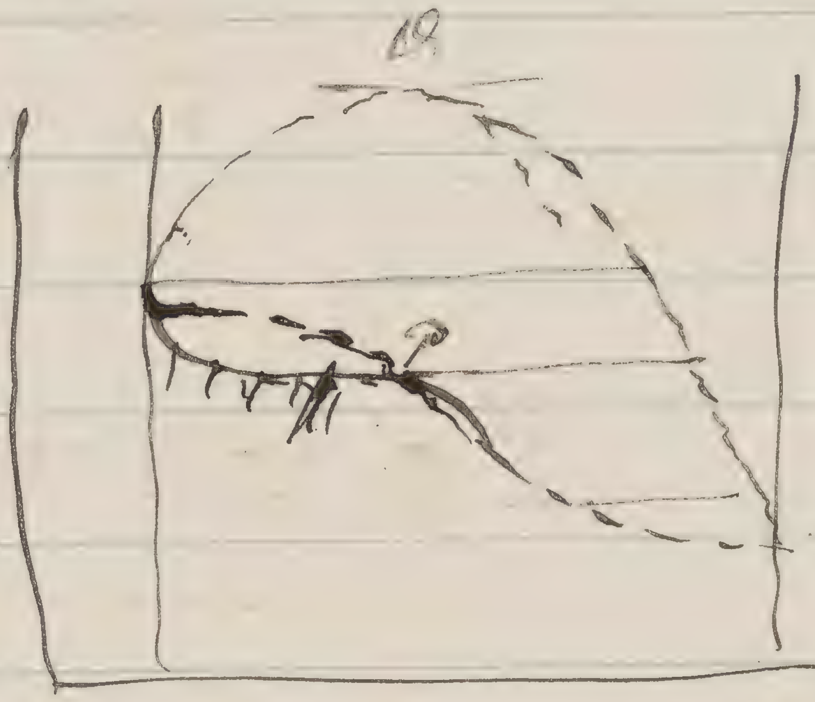
CO_2 en ClH ~~tot Opde~~ kritisch[illegible] CO_2



Oogenblik als de tweede phase [illegible] ontstaan is heeft men het plooipunt de binodale lijn bereikt

in P $\qquad \frac{dp}{dx_2}\left\{\frac{v_2-v_1}{x_2-x_1} - \left(\frac{\partial v_2}{\partial x_2}\right)_p\right\} = 0$

Dus in P $\frac{dp}{dx_2} = 0$

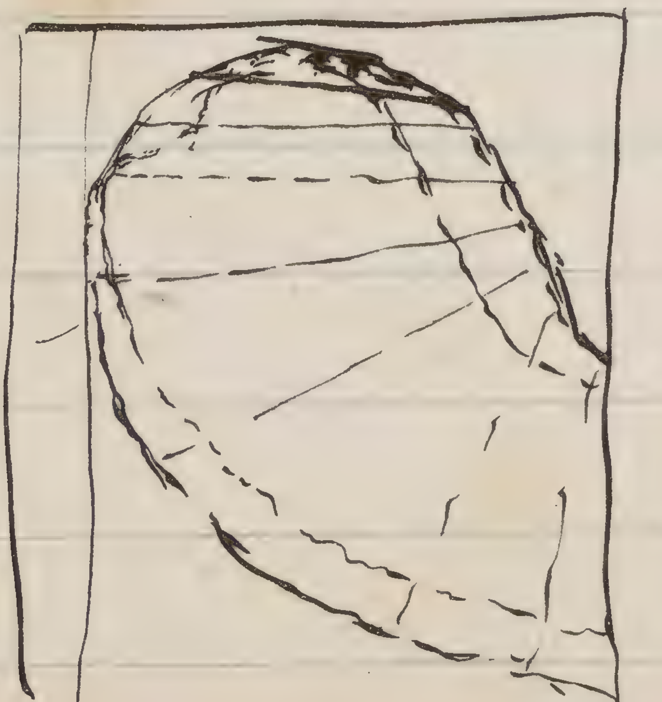


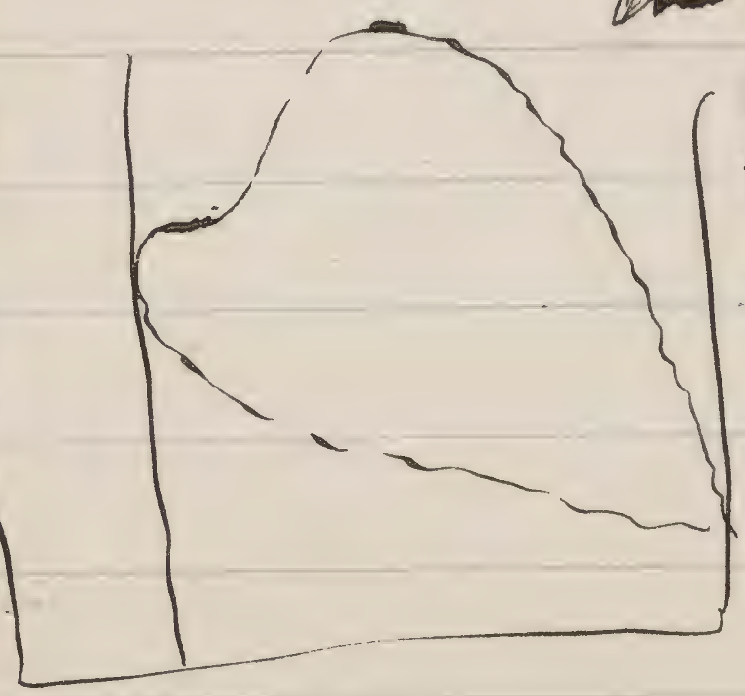
ligt P zoo [illegible] (bij lagere T) nog meer links beginnen, dan komt in T' [illegible]

en in T komen

Dus heeft [illegible] het gebied grooter der 3 coexisterende phasen. +

+ Zonder druk verhooging [illegible] 2 (Kent) grens der [illegible]?



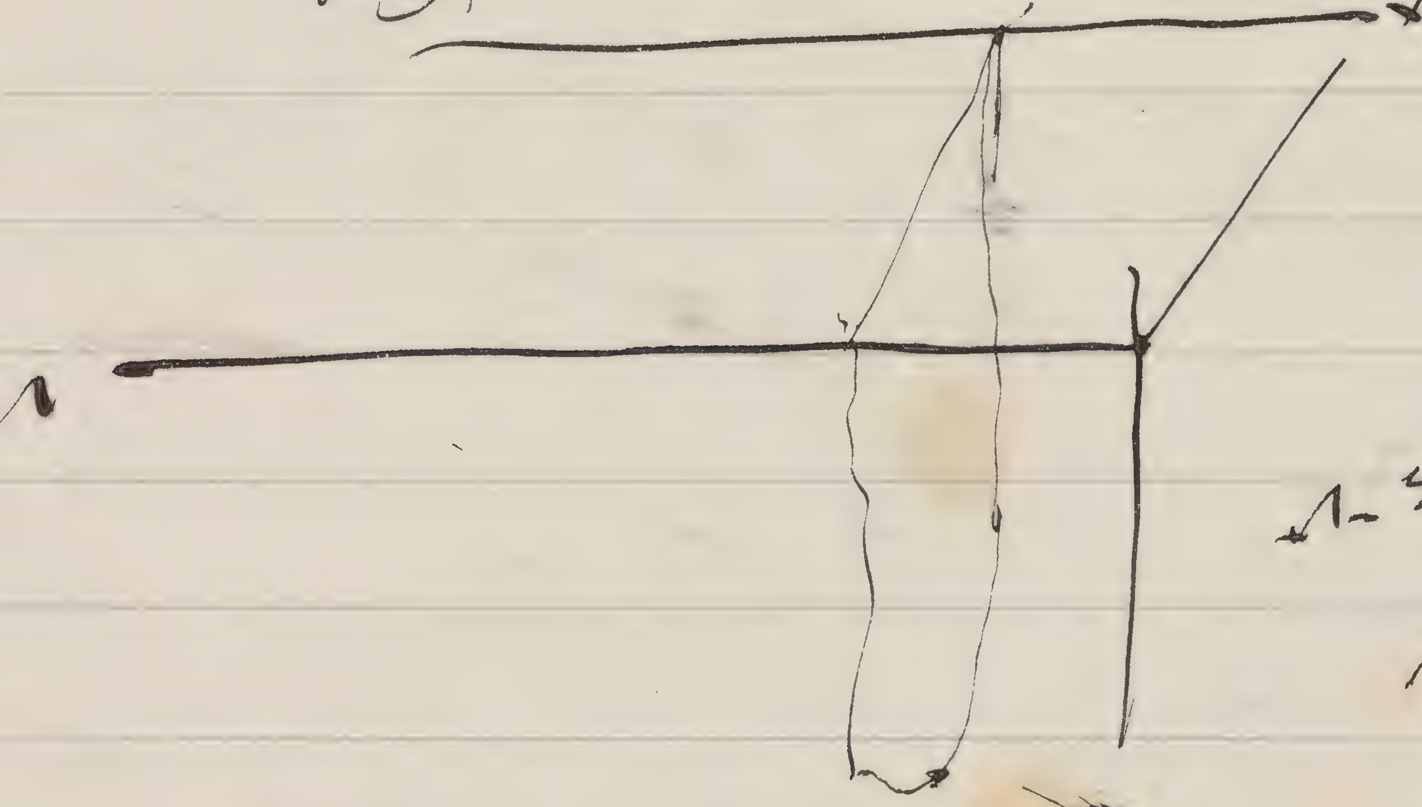


Bij drukbeschouwingen [illegible] constantes te spreken

$\pi = +\int p\,dv$

als functie van x en v

$= -\frac{MRT}{m}\log(v-b) + \frac{a}{v}$



$\mu_2 - \mu_1 = MRT\log\left(\frac{x}{1-x}\right) - \int\frac{\partial p}{\partial x}dv = MRT\log\frac{x}{1-x} - \left(\frac{\partial \pi}{\partial x}\right)_v$

$\left(\frac{\partial^2\zeta}{\partial x^2}\right)_{pT} = \frac{MRT}{x(1-x)} - \left\{\left(\frac{\partial^2\pi}{\partial x^2}\right)_v + \frac{\partial^2\pi}{\partial x\,\partial v}\left(\frac{dv}{dx}\right)_p\right\} \qquad \frac{dv}{dx} = -\frac{\left(\frac{\partial p}{\partial x}\right)_v}{\left(\frac{\partial p}{\partial v}\right)_x}$

$\left(\frac{\partial^2\zeta}{\partial x^2}\right)_{pT} = \frac{MRT}{x(1-x)} - \left\{\left(\frac{\partial^2\pi}{\partial x^2}\right)_v - \frac{\left(\frac{\partial^2\pi}{\partial x\,\partial v}\right)^2}{\frac{\partial^2\pi}{\partial v^2}}\right\}$

Zoolang $\frac{\partial^2\pi}{\partial x^2} - \frac{\left(\frac{\partial^2\pi}{\partial x\,\partial v}\right)^2}{\frac{\partial^2\pi}{\partial v^2}} > 0$ is $\left(\frac{\partial^2\zeta}{\partial x^2}\right)_{pT}$ voor $x = x_2^v < \frac{\partial^2\zeta}{\partial x_2^2}$

$\frac{\partial^2\pi}{\partial x^2}dx^2 + 2\frac{\partial^2\pi}{\partial x\,\partial v}dx\,dv + \frac{\partial^2\pi}{\partial v^2}dv^2$

$\frac{dx}{dv} = -\frac{\frac{\partial^2\pi}{\partial x\,\partial v}}{\frac{\partial^2\pi}{\partial x^2}} \pm \sqrt{\left\{\frac{\left(\frac{\partial^2\pi}{\partial x\,\partial v}\right)^2 - [illegible]}{\left(\frac{\partial^2\pi}{\partial x^2}\right)^2}\right\}}$

Dus om [illegible] het oppervlak met

bij minimumdruk is $\frac{\partial^2\psi}{\partial x^2} \gtrless \frac{\partial^2\psi}{\partial x\,\partial v}$ enz.

is $-\frac{\partial^2 a}{\partial x^2}\frac{1}{v_1} + \frac{MRT}{(v-b)^2}\left(\frac{\partial b}{\partial x}\right)^2 - \frac{\left(\frac{MRT}{(v-b)^2}\frac{\partial b}{\partial x} - \frac{\partial a}{\partial x}\frac{1}{v^2}\right)^2}{\frac{MRT}{(v-b)^2} - \frac{2a}{v^3}} > 0$

of $-\frac{\partial^2 a}{\partial x^2}\frac{1}{v_1} + \frac{\left\{-\frac{MRT}{(v-b)^2}\left(\frac{\partial b}{\partial x}\right)^2\frac{2a}{v^3} + 2\frac{MRT}{v^2(v-b)^2}\frac{\partial b}{\partial x}\frac{\partial a}{\partial x} - \left(\frac{\partial a}{\partial x}\right)^2\frac{1}{v^4}\right\}}{\frac{MRT}{(v-b)^2} - \frac{2a}{v^3}} \gtrless 0$

$\frac{MRT}{(v-b)^2}\frac{\partial b}{\partial x} = \frac{\partial a}{\partial x}\frac{1}{v^2}$

$-\frac{\partial^2 a}{\partial x^2}\frac{1}{v_1} + \frac{2\frac{MRT}{(v-b)^2}\frac{\partial b}{\partial x}\frac{1}{v^2}\left\{-\frac{a}{v^2}\frac{\partial b}{\partial x} + \frac{1}{v}\frac{\partial a}{\partial x}\right\} - \left(\frac{\partial a}{\partial x}\right)^2\frac{1}{v^4}}{\frac{MRT}{(v-b)^2} - \frac{2a}{v^3}} \gtrless 0$

$-\frac{\partial^2 a}{\partial x^2}\frac{1}{v_1} + \frac{2\frac{MRT}{(v-b)^2}\frac{\partial b}{\partial x}\frac{1}{v}\frac{\partial b}{\partial x}\Big| 0 - \left(\frac{\partial a}{\partial x}\right)^2\frac{1}{v^4}}{\frac{MRT}{(v-b)^2} - \frac{2a}{v^3}} > 0$

$\frac{\partial \psi}{\partial x}$ zeer klein

$-\frac{\partial^2 a}{\partial x^2}\frac{1}{v} - \frac{\left(\frac{\partial a}{\partial x}\right)^2\frac{1}{v^4}}{\frac{MRT}{(v-b)^2} - \frac{2a}{v^3}} > 0$ $\quad \frac{\partial b}{\partial x} = \frac{a}{v}\frac{\partial b}{\partial x}$

of $2(2a_{12} - a_1 - a_2) > \frac{\left(\frac{\partial a}{\partial x}\right)^2\frac{1}{v^3}}{-\left(\frac{\partial p}{\partial v}\right)}$

of $-\frac{\partial^2 a}{\partial x^2}\left(\frac{1}{MRT}\frac{a^2}{v^2} - \frac{2a}{v}\right) - \left(\frac{\partial a}{\partial x}\right)^2\frac{1}{v} > 0$

$-\frac{\partial^2 a}{\partial x^2}\frac{1}{MRT}\frac{a^2}{v^2} + \left\{2a\frac{\partial^2 a}{\partial x^2} - \left(\frac{\partial a}{\partial x}\right)^2\right\} > 0$

$4(a_1 + a_2 - 2a_{12})\left\{a_1 + 2(a_{12} - a_1)x + (a_1 + a_2 - 2a_{12})x^2\right\}$ =

$-\ 4\left\{(a_{12} - a_1)^2 + 2(a_{12} - a_1)(a_1 + a_2 - 2a_{12})x + (a_1 + a_2 - 2a_{12})^2x^2\right\}$

$4\left\{a_1 a_2 - a_{12}^2\right\}$

---

eindelijk $\frac{(2a_{12} - a_1 - a_2)}{MRT}\frac{a^2}{v} + 2\frac{(a_{12}^2 - a_1 a_2)}{a^2} > 0$

Dus bij minimumdruk is $2a_{12} > a_1 + a_2$

Bij maximumdruk moet $2a_{12} - a_1 - a_2 < 0$ zijn

~~$\frac{2a_{12}}{b_1 + b_2} = \frac{a_1}{b_1} + \ldots$~~

~~$\frac{2a_{12}}{b_1 + b_2} = \frac{a_2}{b_2} + \ldots$~~

~~$2a_{12} = a_1 + a_2 + b_1 l + b_2 m$~~

~~$2a_{12} - a_1 - a_2 = b_1 l + b_2 m$~~

minimumdruk $\frac{\partial a}{\partial x} = \frac{a}{v}\frac{\partial b}{\partial x}$

$2(a_{12} - a_1) + (a_1 + a_2 - 2a_{12})x = \frac{b_2 - b_1}{2v}\left[a_1 + 2(a_{12} - a_1)x + (a_1 + a_2 - 2a_{12})x^2\right]$

$a_{12} - a_1 > \frac{b_2 - b_1}{2v_1}a_1$

$a_{12} > a_1\frac{2v_1 + b_2 - b_1}{2v_1}$ of $\frac{2a_{12}}{2v_1 - b_1 + b_2} > \frac{a_1}{v_1}$ $\quad$ zij $v_1 = n_1 b_1$

en

$a_2 - a_{12} < \frac{b_2 - b_1}{2v_2}a_2$ $\quad$ of $\frac{2a_{12}}{2v_2 - b_2 + b_1} > \frac{a_2}{v_2}$ $\quad v_2 = n_2 b_2$

$a_2\frac{2v_2 - b_2 + b_1}{2v_2} < a_{12}$ $\quad \frac{a_{12}}{\frac{(2n_1 - 1)b_1 + b_2}{2n_1}} > \frac{a_1}{b_1}$

$\frac{a_{12}}{\frac{(2n_2 - 1)b_2 + b_1}{2n_2}} > \frac{a_2}{b_2}$

Verschil

$\frac{(2n_1 - 1)b_1 + b_2}{2n_1} - \frac{(b_1 + b_2)}{2n_1}$

~~$\frac{b_1 - b_2(2n_1 - 1)}{2n_1}$~~

$\frac{(n_1 - 1)}{2n_1}(b_1 - b_2)$

$$\frac{d^2 \frac{a}{b}}{dx^2} = \frac{2\{a_1 b_2^2 + a_2 b_1^2 - 2a_{12} b_1 b_2\}}{b^3}$$

$$a_1 + a_2 - 2a_{12} = \Delta$$

$$a_1 b_2^2 + a_2 b_1^2 - 2a_{12} b_1 b_2 = K$$

$$a_1 b_1 b_2 + a_2 b_1 b_2 - 2a_{12} b_1 b_2 = b_1 b_2 \Delta$$

$$a_1 (b_2 - b_1) b_2 - a_2 (b_2 - b_1) b_1 = K - b_1 b_2 \Delta$$

$$\left(\frac{a_1}{b_1} - \frac{a_2}{b_2}\right)(b_2 - b_1) = \frac{K}{b_1 b_2} - \Delta$$

$$\Delta = \frac{K}{b_1 b_2} - \left(\frac{a_1}{b_1} - \frac{a_2}{b_2}\right)(b_2 - b_1) \quad \text{ondersteld } \frac{a_1}{b_1} > \frac{a_2}{b_2}$$

is K negatief dan ook Δ

Dus als menging warmte geeft is $2a_{12} > a_1 + a_2$

is K positief dan nog niet altijd Δ

Dus het verschijnsel dat mengsel by lage T warmte absorbeert en by hoogere warmte voortbrengt

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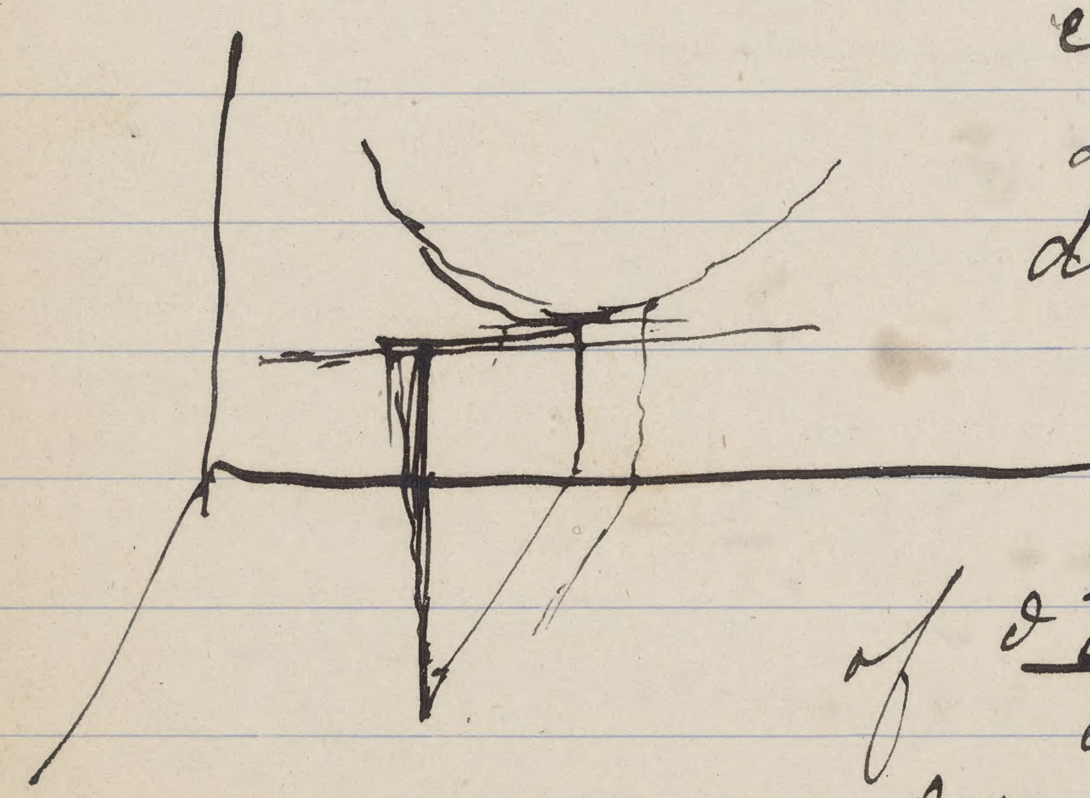

Minimumdruk

Zal x met T toenemen

dan moet $\frac{dp}{dT}$ met x afnemen

of $\frac{dx}{dT}$ positief als $\frac{\partial\left(\frac{\eta_2 - \eta_1}{v_2 - v_1}\right)}{\partial x} < 0$

of $\frac{\partial\left\{p + \frac{a}{v_1 v_2}\right\}}{\partial x} < 0$ of $\frac{\partial^2 a}{\partial x^2} < 0$

Dit wordt by ClH niet bevestigd

DSI

Amsterdam 28 März 1935.

Herrn Dr. Ing A Weiner
Brünn.

Sehr Geehrter Herr Doctor,

Ich habe noch ein MS. meines Vaters gefunden, dass ich Ihnen wohl abtreten kann.

Ergebenst.

J D v d Waals Jr

Tsecho-Slovakije

Dr. Ing. Armin Weiner.
Generaldirector der Westmährischen
Electrizitäts werke, A. G.

Nova 39

Brünn.

Exp. Prof. Dr. J. D. Waals Jr.
Willemsparkweg 204
Amsterdam
Pays-Bas.

AUTOGRAPHE

de

van der Waals

Johannes Diderik

OBSERVATIONS

M.a. 40 p.4o, o.O.u.D., Berechnungen und Studien auf dem Gebiete der Druc[k]linien bei Binären Gemischen, der Fläche ψ bei denselben und insbesondere bei CO_2 UND ClH.

Geschenk seines Sohnes Prof.Dr. J.D. v.d.Waals jr., Prof. der Physik an der Univ. Amsterdam.

Van der Waals, Johannes Diderik, - Physiker.
Geb. 23.11.1837 in Leyden,
gest. 9.3. 1923, Amsterdam.

Stud. Univ. Leyden, war dann Mittelschullehrer in Deventer und Haag u. schrieb 1873 seine erste wissensch. Arbeit ,, Ueber die Kontinuität des gasförmigen und flüssigen Zustandes". Wirkte von 1877 bis 1907 als Prof.d. Physik a.d. Univ. Amsterdam. Schon in seiner ersten Arbeit gelangte er durch die Verbindung der thermodynam. Theorie der Kapillarität mit der kinetischen Theorie der Gase zur Feststellung der Kontinuität und Darstellung einer Zustandsgleichung, welche einheitlich für Gase und Flüssigkeiten gilt; fand 1880 das Gesetz der korrespondierenden Zustände und 1890 die Erklärung des Verhaltens der binären und ternären Gemische mit Hilfe der Zustandsgleichung. Erhielt 1910 den Nobelpreis für seine Arbeiten über die Zustandsgleichung der Gase und Flüssigkeiten.